# 4-8 Magnetic moments

◎ nucleon magnetic moment with isospin

$$\begin{cases} \mu_p = \frac{e}{2M}\left(g_s^{(p)} s + \underset{\substack{\Downarrow \\ g_\ell^{(p)}=1}}{g_\ell^{(p)}} \ell\right) \\ \mu_n = \frac{e}{2M}\left(g_s^{(n)} s + \underset{\substack{\| \\ 0}}{g_\ell^{(n)}} \ell\right) \end{cases}$$

$$\begin{cases} g_s^{(p)} = 5.5855 \\ g_s^{(n)} = -3.8263 \end{cases}$$

g-factor is different from $\underline{g=2}$

$\Uparrow$ nucleon is not a point particle!!

$$\mu = \frac{1}{2M}\left[\frac{1}{2}\left(g_s^{(p)} + g_s^{(n)}\right) s + \frac{1}{2}\left(g_s^{(p)} - g_s^{(n)}\right)\tau_z s + \frac{1}{2}(1+\tau_z)\, g_\ell^{(p)} \ell\right]$$

$$\therefore \boxed{\mu = \frac{e}{2M}\left[0.8796\, s + 4.706\, \tau_3\, s + 0.5\,(1+\tau_3)\,\ell\right]}$$

# [ Nucleon magnetic moments ]

## Quark model

| | $u$ | $d$ | $s$ | |
|---|---|---|---|---|
| $Q$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | (charge) |
| $I_3$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $0$ | (isospin) |
| $S$ | $0$ | $0$ | $-1$ | (strangeness) |
| $B$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | (Baryon number) |

∘ $\begin{cases} \text{proton} : uud \\ \text{neutron} : udd \end{cases}$ $\qquad \begin{cases} u : \frac{2}{3}e \\ d : -\frac{1}{3}e \end{cases}$

$\Uparrow$

(electric charge)

◎ magnetic moment operator

$$\boxed{\mu_z = \mu_0 \sum_{i=1}^{3} e_i \, \sigma_i^z} \qquad \left( \mu_0 : \text{some scale} \right)$$

$$\left( e_u = \frac{2}{3}e \;,\; e_d = -\frac{1}{3}e \right)$$

# [ Proton wave function ]
## (SU(6) quark model)

Proton spin wave function $|P\rangle_\uparrow$

$$\boxed{\begin{aligned}|P\rangle_\uparrow = \frac{1}{\sqrt{18}}\Big[ & 2u^{(1)}_\uparrow d^{(2)}_\downarrow u^{(3)}_\uparrow + 2u^{(1)}_\uparrow u^{(2)}_\uparrow d^{(3)}_\downarrow + 2d^{(1)}_\downarrow u^{(2)}_\uparrow u^{(3)}_\uparrow \\ & - u^{(1)}_\uparrow u^{(2)}_\downarrow d^{(3)}_\uparrow - u^{(1)}_\uparrow d^{(2)}_\uparrow u^{(3)}_\downarrow - u^{(1)}_\downarrow d^{(2)}_\uparrow u^{(3)}_\uparrow \\ & - d^{(1)}_\uparrow u^{(2)}_\downarrow u^{(3)}_\uparrow - d^{(1)}_\uparrow u^{(2)}_\uparrow u^{(3)}_\downarrow - u^{(1)}_\downarrow u^{(2)}_\uparrow d^{(3)}_\uparrow \Big]\end{aligned}}$$

(This is obtained by making up the SU(6) group theory)

⊛ Magnetic moment of proton :

$$\langle P_\uparrow | \mu_z | P_\uparrow \rangle = \mu_0 \langle P_\uparrow | \sum_{i=1}^{3} e_i \sigma_i^z | P_\uparrow \rangle$$

Example①

$$\langle u^{(1)}_\uparrow d^{(2)}_\downarrow u^{(3)}_\uparrow | \sum_{i=1}^{3} e_i \sigma_i^z | u^{(1)}_\uparrow d^{(2)}_\downarrow u^{(3)}_\uparrow \rangle$$

$$= e_1 \langle u^{(1)}_\uparrow | \sigma_1^z | u^{(1)}_\uparrow \rangle \langle d^{(2)}_\downarrow | d^{(2)}_\downarrow \rangle \langle u^{(3)}_\uparrow | u^{(3)}_\uparrow \rangle$$

$$+ e_2 \langle d^{(2)}_\downarrow | \sigma_2^z | d^{(2)}_\downarrow \rangle \langle u^{(1)}_\uparrow | u^{(1)}_\uparrow \rangle \langle u^{(3)}_\uparrow | u^{(3)}_\uparrow \rangle$$

$$+ e_3 \langle u^{(3)}_\uparrow | \sigma_3^z | u^{(3)}_\uparrow \rangle \langle u^{(1)}_\uparrow | u^{(1)}_\uparrow \rangle \langle d^{(2)}_\downarrow | d^{(2)}_\downarrow \rangle$$

$$= \frac{2}{3}e + \left(-\frac{1}{3}\right)(-e) + \frac{2}{3}e = \underline{\frac{5}{3}e}$$

Example ② No cross term:

$$\langle u_\uparrow^{(1)} u_\uparrow^{(2)} d_\downarrow^{(3)} | \sum_{i=1}^{3} e_i \sigma_i^z | u_\uparrow^{(1)} d_\downarrow^{(2)} u_\uparrow^{(3)} \rangle$$

$$= 0 \qquad \text{(orthogonal to each other)}$$

Therefore, we obtain

$$\langle P_\uparrow | \mu_p^z | P_\uparrow \rangle$$

$$= \frac{\mu_0}{18} \left[ 4 \left( \frac{2}{3} + \frac{1}{3} + \frac{2}{3} \right) \times 3 + \left( \frac{2}{3} - \frac{2}{3} - \frac{1}{3} \right) \times 6 \right]$$

$$= \frac{\mu_0}{18} \left[ \frac{5}{3} \times 4 \times 3 - \frac{6}{3} \right] = \mu_0$$

$$\boxed{\mu_p = \mu_0}$$

# [ Neutron wave function ]

$$
\begin{aligned}
|n_\uparrow\rangle = \frac{1}{\sqrt{18}} \Big[ & 2\, d_\uparrow^{(1)} u_\downarrow^{(2)} d_\uparrow^{(3)} + 2\, d_\uparrow^{(1)} d_\uparrow^{(2)} u_\downarrow^{(3)} + 2\, u_\downarrow^{(1)} d_\uparrow^{(2)} d_\uparrow^{(3)} \\
& - d_\uparrow^{(1)} d_\downarrow^{(2)} u_\uparrow^{(3)} - d_\uparrow^{(1)} u_\uparrow^{(2)} d_\downarrow^{(3)} - d_\downarrow^{(1)} u_\uparrow^{(2)} d_\uparrow^{(3)} \\
& - u_\uparrow^{(1)} d_\uparrow^{(2)} d_\downarrow^{(3)} - u_\uparrow^{(1)} d_\downarrow^{(2)} d_\uparrow^{(3)} - d_\downarrow^{(1)} d_\uparrow^{(2)} u_\uparrow^{(3)} \Big]
\end{aligned}
$$

- Magnetic moment of neutron :

$$\mu_n = \mu_0 \langle n_\uparrow | \sum_{i=1}^{3} e_i \sigma_i^z | n_\uparrow \rangle$$

$$= \frac{\mu_0}{18} \left[ 4\left(-\frac{2}{3} - \frac{2}{3}\right) \times 3 + 6\left(\frac{2}{3} - \frac{1}{3} + \frac{1}{3}\right) \right]$$

$$\therefore \boxed{\mu_n = -\frac{2}{3}\mu_0}$$

## [ Comparison of Theory with Experiment ]

$$\boxed{\begin{aligned} \mu_p &= \mu_0 \\ \mu_n &= -\frac{2}{3}\mu_0 \end{aligned}}$$

$$\left(\frac{\mu_p}{\mu_n}\right)_{th} = -1.5 \quad \text{(theory)}$$

$$\left(\frac{\mu_p}{\mu_n}\right)_{exp} \simeq -1.46 \quad \text{(experiment)}$$

The agreement is excellent !!