Evaluation of Triangle Diagrams

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Abstract. We present a technical report of the careful calculation of the triangle diagrams which are originally evaluated by Adler in 1969. First, we confirm that Nishijima's method of the $\pi^0 \to \gamma+\gamma$ calculation with γ^5 vertex can precisely reproduce the observed life time of π^0 decay. Then, we calculate the T-matrix of the $Z^0 \to \gamma + \gamma$ process in which the vertex of the $\gamma^\mu\gamma_5$ is responsible for the decay of the weak vector boson. Even though the decay rate itself vanishes to zero due to the symmetry nature of two photons (Landau-Yang theorem), the T-matrix of the process has neither linear nor logarithmic divergences. All the triangle diagrams are finite and consistent with experiments, and this is all what we should understand in theoretical physics.

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1. Introduction

In 1969, Nishijima presented the calculation of the π^0 decay process quite in detail in the field theory textbook [1], and his calculation of the decay process of $\pi^0 \to 2\gamma$ is based on the standard perturbation calculation. The calculated result of the decay width is given as

$$\Gamma_{\pi^0\to 2\gamma} = \frac{\alpha^2}{16\pi^2}\frac{g_\pi^2}{4\pi}\left(\frac{m_\pi}{M_N}\right)^2 m_\pi \tag{1.1}$$

where α, m_π and M_N denote the fine structure constant, the mass of pion and the mass of nucleon, respectively. Therefore, there is no anomaly in the T-matrix evaluation of $\pi^0 \to 2\gamma$ if one carries out the calculation properly. Here, there is no ambiguity of the T-matrix evaluation since all of the divergences vanish to zero at the level of Trace evaluations.

For a long time, people believe that the decay rate of $\pi^0 \to 2\gamma$ should be described in terms of the anomaly equation which is proposed by Adler in 1969 [2]. This is somewhat surprising since the decay rate of $\pi^0 \to 2\gamma$ is well described by the calculation of Nishijima in terms of the standard Feynman diagrams before the work of Adler.

However, Adler claimed in 1969 that the T-matrix of the two photon decay with the axial vector coupling $\gamma^\mu\gamma^5$ should be regularized because of the linear divergence, and then he derived the anomaly equation. In his paper, however, he assumed that the Feynman diagram of two photons interchanged (the second diagram) should be just the same as the first diagram, and therefore he just multiplied twice the first diagram in his calculation.

However, the sum of the two diagrams should vanish to zero due to the Landau-Yang theorem, and thus Adler's calculation disagrees with the theorem even though he knew the theorem as a knowledge. To be more specific, one sees that the spin states constructed by two photons should be three, namely, rank 0 (scalar), rank 1 (vector) and rank 2 tensor states, respectively, in terms of the group theory terminology, and this is clear since the polarization vector is a rank one tensor. In this case, one can easily prove that the rank 0 tensor should be symmetric between two photon polarization vectors while rank one tensor should be anti-symmetric under the exchange of $1 \leftrightarrow 2$. Therefore, it is clear that the two photon states cannot make the spin 1 state since two photons should be physically symmetric. This is the essence of the Landau-Yang theorem.

In this paper, we first confirm that the decay rate of $\pi^0 \to 2\gamma$ can be indeed described by Nishijima's calculation. The interaction between pions and nucleons can be described in terms of the pseudoscalar coupling as

$$\mathcal{L}_I = ig_\pi \bar{\psi}\gamma_5 \boldsymbol{\tau}\psi \cdot \boldsymbol{\varphi} \tag{1.2}$$

where g_π denotes the pion-nucleon coupling constant. If we take the value of the pion-nucleon coupling constant g_π as $\frac{g_\pi^2}{4\pi} \simeq 8$ which is slightly smaller than the value as suggested from the nucleon-nucleon scattering data [3], we obtain the decay width of $\Gamma_{\pi^0\to 2\gamma} \simeq 7.5$ eV which should be compared with the observed value of 7.8 eV. Note that the πNN coupling constant in the pion decay should include the form factor effect of the $N-N$ scattering case, and thus it should be smaller than the scattering value.

In the same way, we can calculate the decay rate of the $Z^0 \to 2\gamma$ process. In this case, the interaction Lagrangian density for the Z^0 boson Z^μ and fermions ψ_ℓ can be written as [4]

$$\mathcal{L}_{II} = g_z \bar{\psi}_\ell \gamma^\mu \gamma_5 \psi_\ell Z_\mu - 0.06 g_z \bar{\psi}_\ell \gamma^\mu \psi_\ell Z_\mu \tag{1.3}$$

which is obtained from the standard model weak Hamiltonian with $\sin^2\theta_W = 0.235$. Here, the first term in eq.(1.3) is important since the decay of the Z^0 boson into two photons is described by the parity violating part. In this case, the decay rate of the $Z^0 \to 2\gamma$ can be described by the triangle diagrams which are basically the same as the π^0 decay process. The T-matrix of the $Z^0 \to 2\gamma$ process has neither linear nor logarithmic divergence due to the Trace and parameter integrals. In particular, one can easily prove that the linear divergent part should vanish to zero by making use of the following identity equation

$$\mathrm{Tr}\left\{\not{p}\gamma_\mu \not{p}\gamma_\nu \not{p}\gamma_\rho \gamma_5\right\} = -\mathrm{Tr}\left\{\not{p}\gamma_\nu \not{p}\gamma_\mu \not{p}\gamma_\rho \gamma_5\right\}.$$

Therefore the total T-matrix of the $Z^0 \to 2\gamma$ process is given as

$$T_{Z^0\to 2\gamma} = -\frac{g_z}{6\pi^2}\left(\frac{2e}{3}\right)^2 (k_1^\alpha - k_2^\alpha)\varepsilon_{\mu\nu\rho\alpha}\epsilon_1^\mu \epsilon_2^\nu \epsilon_v^\rho \tag{1.4}$$

where $(k_1^\mu, \epsilon_1^\mu)$ and $(k_2^\mu, \epsilon_2^\mu)$ denote the four momentum and polarization vector of two photons, respectively, and ϵ_v^ρ denotes the polarization vector of Z^0 boson. As we see below, this T-matrix is shown to vanish to zero

$$T_{Z^0\to 2\gamma} = 0 \tag{1.5}$$

due to the symmetry nature of the two photon states, which is consistent with Landau-Yang theorem [5, 6]. Therefore, all of the physical observables related to the triangle diagrams can be properly reproduced, and thus the present calculation rigorously proves that physically there is no room for the anomaly equation.

2. $\pi^0 \to \gamma + \gamma$ process

Before going to the discussion of the $Z^0 \to 2\gamma$ decay, we first review the calculation of the $\pi^0 \to 2\gamma$ process which is first given by Nishijima [1]. The interaction Lagrangian density $\mathcal{L}_I$ between fermion and pion can be given as eq.(1.2). In this case, the corresponding T-matrix for the $\pi^0 \to 2\gamma$ reaction process can be written as

$$T_{\pi^0\to 2\gamma} = ie^2 g_\pi \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\gamma\epsilon_1)\frac{1}{\not{p} - M_N + i\varepsilon}(\gamma\epsilon_2)\frac{1}{\not{p} - \not{k}_2 - M_N + i\varepsilon}\gamma_5 \frac{1}{\not{p} + \not{k}_1 - M_N + i\varepsilon}\right]$$
$$+(1 \leftrightarrow 2). \tag{2.1}$$

where $\epsilon_1^\mu(\lambda_1)$ and $\epsilon_2^\mu(\lambda_2)$ denote the two polarization vectors of photons with the polarizations of $\lambda_1,\ \lambda_2$. In addition, $k_1^\mu,\ k_2^\mu$ denote the four momenta of two photons. Now, we can rewrite eq.(2.1) to evaluate the Trace parts as

$$T_{\pi^0\to 2\gamma} = 2e^2 g_\pi \int \frac{d^4p}{(2\pi)^4}\ \frac{A_{\mu\nu}\epsilon_1^\mu \epsilon_2^\nu}{(p^2-M^2)((p-k_2)^2-M^2)((p+k_1)^2-M^2)} \tag{2.2}$$

where $A_{\mu\nu}$ is defined as

$$A_{\mu\nu} \equiv \mathrm{Tr}[\gamma_\mu(\not p + M)\gamma_\nu(\not p - \not k_2 + M)\gamma^5(\not p + \not k_1 + M)]. \tag{2.3}$$

2.1. Linear Divergence Term

Now the linear divergence term should correspond to the term which is proportional to p^3 in eq.(2.3), and thus we can show

$$A_{\mu\nu}^{(3)} = \mathrm{Tr}[\not p\gamma_\mu \not p \gamma_\nu \not p \gamma^5] = 0 \tag{2.4}$$

which is due to the property of the Trace with γ^5 matrix.

2.2. Logarithmic Divergence Term

Next, we should evaluate the p^2 term in eq.(2.3) which should correspond to the logarithmic divergence term. Now, this term can be written as

$$A_{\mu\nu}^{(2)} = \mathrm{Tr}[\gamma_\mu \not p \gamma_\nu \not p \gamma^5] + \mathrm{Tr}[\not p \gamma_\mu \gamma_\nu \not p \gamma^5] + \mathrm{Tr}[\not p \gamma_\mu \not p \gamma_\nu \gamma^5] = 0 \tag{2.5}$$

and it also vanishes to zero by the Trace evaluation. Here we have made use of the following identity

$$\mathrm{Tr}[\gamma_\mu\ \not p\ \gamma_\nu\ \not p \gamma^5] = -4i\varepsilon_{\mu\rho\nu\sigma}p^\rho p^\sigma = 0.$$

Therefore, the T-matrix of the $\pi^0 \to 2\gamma$ process has neither linear nor logarithmic divergences, and this is proved at the level of the Trace evaluation before the momentum integrations.

2.3. Finite Term

Now, we can easily evaluate this momentum integral, and the result becomes

$$T_{\pi^0\to 2\gamma} \simeq \frac{e^2 g_\pi}{4\pi^2 M}\varepsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \epsilon_1^\mu \epsilon_2^\nu. \tag{2.6}$$

As one sees, there is no divergence in this T-matrix calculation, and this is because the apparent linear and logarithmic divergences can be completely canceled out due to the Trace evaluation. In this respect, the corresponding T-matrix is finite and thus there is no chiral anomaly in this Feynman diagrams. This is, of course, well known, and the calculation of the T-matrix is explained quite in detail in the textbook of Nishijima in 1969 [1].

2.4. Decay Width of $\pi^0 \to 2\gamma$

In this case, we can calculate the decay width $\Gamma_{\pi^0\to 2\gamma}$ as

$$\Gamma_{\pi^0\to 2\gamma} = \frac{1}{8m_\pi|\boldsymbol{p}_1||\boldsymbol{p}_2|(2\pi)^2}\int \delta(m_\pi - |\boldsymbol{p}_1| - |\boldsymbol{p}_2|)\delta(\boldsymbol{p}_1+\boldsymbol{p}_2)|U|^2 d^3p_1 d^3p_2 \tag{2.7}$$

where $|U|^2$ is given as

$$|U|^2 = \frac{1}{2}\sum_{\lambda_1,\lambda_2} |T_{\pi^0\to 2\gamma}|^2 \tag{2.8}$$

where λ_1 and λ_2 denote the polarization state of two photons. The summation of the polarization state of two photons can be carried out by making use of the Coulomb gauge fixing which gives the polarization sum as

$$\sum_{\lambda=1}^{2} \epsilon^{*\mu}_{\boldsymbol{k},\lambda}\epsilon^{\nu}_{\boldsymbol{k},\lambda} = \begin{cases} \left(\delta^{ab} - \frac{k^a k^b}{\boldsymbol{k}^2}\right) & \text{for} \quad \mu \neq 0, \nu \neq 0 \\ \\ 0 & \text{for} \quad \mu, \nu = 0. \end{cases} \tag{2.9}$$

After some calculations, we obtain the decay width $\Gamma_{\pi^0\to 2\gamma}$ as

$$\Gamma_{\pi^0\to 2\gamma} \simeq \frac{\alpha^2}{16\pi^2}\frac{g_\pi^2}{4\pi}\frac{m_\pi^3}{M^2} \simeq 7.4 \quad \text{eV} \tag{2.10}$$

which can be compared with the observed value [7]

$$\Gamma^{exp}_{\pi^0\to 2\gamma} = 7.8 \ \ \text{eV}. \tag{2.11}$$

As seen above, the calculation can well reproduce the observed data of the life time of the $\pi^0 \to 2\gamma$ decay. Here, we take the value of $\frac{g_\pi^2}{4\pi} \simeq 8$ which should be slightly smaller than the one determined from the nucleon-nucleon scattering experiments. This is clear since the $\pi^0 \to 2\gamma$ process should naturally include the effect of the nucleon form factor, in contrast to the value of the πNN coupling constant obtained from the nucleon-nucleon scattering experiments. In the case of NN scattering, the nucleon form factors are introduced to accommodate the finite size effect of nucleons in the scattering process.

Here, it should be important to note that the calculation of the decay width with the Coulomb gauge fixing is quite involved [8]. On the other hand, the choice of the polarization sum of two photons

$$\sum_{\lambda} \epsilon^{*\mu}_{\boldsymbol{k},\lambda}\epsilon^{\nu}_{\boldsymbol{k},\lambda} = -g^{\mu\nu} \tag{2.12}$$

can also reproduce the correct decay width as given in eq.(2.9). In addition, the calculation with this choice of the polarization sum is much easier than the case with the correct expression of the polarization sum. However, the expression of eq.(2.12) cannot be justified for $\nu = \mu = 0$ case since the left hand side $\sum_\lambda |\epsilon^0_{\boldsymbol{k},\lambda}|^2$ is always positive definite while the right hand side is negative. In this respect, the employment of eq.(2.12) is accidentally justified because of the special property of the amplitude in eq.(2.7).

3. $Z^0 \to \gamma + \gamma$ process

Now we can calculate the triangle Feynman diagrams which correspond to the Z^0 decay into two photons. The interaction Lagrangian density between Z^0 and fermions can be given in eq.(1.3), and therefore, the corresponding T-matrix for the triangle diagrams can be written as

$$T_{Z^0 \to 2\gamma} = \sum_i g_z e_i^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not p - m_i + i\varepsilon}(\gamma\epsilon_1)\frac{1}{\not p - \not k_2 - m_i + i\varepsilon}(\gamma\epsilon_2)\times \right.$$
$$\left. \frac{1}{\not p + \not k_1 - m_i + i\varepsilon}(\gamma\epsilon_v)\gamma_5\right] + (1 \leftrightarrow 2) \tag{3.1}$$

where m_i and e_i denote the mass and charge of the corresponding fermions in the intermediate states.

At this point, we should make a comment on Adler's calculation why he obtained the linear divergence in his paper. This is obviously connected to the fact that he thought that the second term $(1 \leftrightarrow 2)$ should be just the same as the first diagram, and therefore he simply multiplied twice the second term in his calculation of the T-matrix as if it were the $\pi^0 \to 2\gamma$ case in which the two diagrams become just the same due to the symmetry nature. Therefore he could not get rid of the linear divergence in his T-matrix evaluation [2] even though he knew the Landau-Yang theorem.

Here, we take the top quark state as the intermediate state since it gives the largest contribution to the decay width. The evaluation of the T-matrix can be carried out in a straight forward way just in the same manner as the $\pi^0 \to 2\gamma$ process. In order to avoid any confusions, we discuss the term by term in the integration of eq.(3.1).

3.1. Linear Divergence Term

The leading term in the integration of eq.(3.1) at the large momentum of p should have the following shape

$$T^{(1)}_{Z^0 \to 2\gamma} \simeq g_z e^2 \int \frac{d^4p}{(2\pi)^4}\left[\frac{\mathrm{Tr}\left\{\not p\gamma_\mu \not p \gamma_\nu \not p \gamma_\rho \gamma_5\right\} \epsilon_1^\mu \epsilon_2^\nu \epsilon_v^\rho}{(p^2 - s_0 + i\epsilon)^3} + (1 \leftrightarrow 2)\right]. \tag{3.2}$$

In this case, we can easily prove the following equation

$$\mathrm{Tr}\left\{\not p\gamma_\mu \not p \gamma_\nu \not p \gamma_\rho \gamma_5\right\} = -\mathrm{Tr}\left\{\not p\gamma_\nu \not p \gamma_\mu \not p \gamma_\rho \gamma_5\right\}$$

and we obtain

$$T^{(1)}_{Z^0 \to 2\gamma} \simeq g_z e^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2 - s_0 + i\epsilon)^3}\left[\mathrm{Tr}\left\{\not p\gamma_\mu \not p \gamma_\nu \not p \gamma_\rho \gamma_5\right\} + \mathrm{Tr}\left\{\not p\gamma_\nu \not p \gamma_\mu \not p \gamma_\rho \gamma_5\right\}\right]\epsilon_1^\mu \epsilon_2^\nu \epsilon_v^\rho = 0. \tag{3.3}$$

Thus, there is no linear divergence in the triangle diagrams, and thus no need of the regularization [2, 9, 10].

3.2. Logarithmic Divergence Term

The p^2 term of the numerator in eq.(3.1) contains the apparent logarithmic divergence. However, we find that the logarithmic divergence term vanishes to zero in an exact fashion. First, we can calculate the Trace of the $\gamma-$ matrices and find the following shape for the logarithmic divergence term $T^{(0)}_{Z^0\to 2\gamma}$ as

$$T^{(0)}_{Z^0\to 2\gamma} \simeq g_z e^2 \int_0^1 dx \int_0^x dy \int \frac{d^4p}{(2\pi)^4} \frac{F(p,x,y)}{(p^2 - s_0 + i\epsilon)^3} \tag{3.4}$$

where $F(p,x,y)$ is written

$$F(p,x,y) = \mathrm{Tr}\left\{\not p\gamma_\mu \not p\gamma_\nu \not a\gamma_\rho\gamma_5\right\} + \mathrm{Tr}\left\{\not p\gamma_\mu \not b\gamma_\nu \not p\gamma_\rho\gamma_5\right\} + \mathrm{Tr}\left\{\not c\gamma_\mu \not p\gamma_\nu \not p\gamma_\rho\gamma_5\right\} \tag{3.5}$$

where a, b, c are given as

$$a = -k_1(1-x) - k_2(1-y), \quad b = -k_1(1-x) + k_2 y, \quad c = -k_1 x + k_2 y$$

After some tedious but straight forward calculation, we find that

$$T^{(0)}_{Z^0\to 2\gamma} = 0 \tag{3.6}$$

and therefore there is no need of the renormalization since the triangle diagrams are indeed all finite.

3.3. Finite Terms

Here, one sees that the triangle diagrams with the axial vector coupling have neither linear nor logarithmic divergences. This is proved without any regularizations, and the total amplitude of $Z^0 \to 2\gamma$ decay process is indeed finite. Here, we present the calculated decay width via top quarks since its contribution is the largest among all the other fermions. The finite term of the T-matrix can be written as

$$T_{Z^0\to 2\gamma} = g_z e^2 \int_0^1 dx \int_0^x dy \int \frac{d^4p}{(2\pi)^4} \frac{A(x,y)}{(p^2 - s_0 + i\epsilon)^3} \tag{3.7}$$

where $A(x,y)$ is given as

$$A(x,y) = -4im_t^2(x+1-y)(k_1^\alpha - k_2^\alpha)\varepsilon_{\mu\nu\rho\alpha}\epsilon_1^\mu\epsilon_2^\nu\epsilon_v^\rho.$$

Here m_t denotes the mass of the top quark. Therefore, the T-matrix becomes

$$T_{Z^0\to 2\gamma} = -\frac{g_z}{6\pi^2}\left(\frac{2e}{3}\right)^2 (k_1^\alpha - k_2^\alpha)\varepsilon_{\mu\nu\rho\alpha}\epsilon_1^\mu\epsilon_2^\nu\epsilon_v^\rho \tag{3.8}$$

Now, we can prove that this should vanish to zero by choosing the system where Z^0 boson should be at rest. In this case, we can take the polarization vector ϵ_v^ρ as

$$\epsilon_v^\rho = (0, \boldsymbol{\epsilon}_v) \tag{3.9}$$

which can satisfy the Lorentz condition of $k_\mu \epsilon_v^\mu = 0$. On the other hand, we can also choose the photon polarization vectors ϵ_1^μ and ϵ_1^ν with the Coulomb gauge fixing as

$$\epsilon_1^\mu = (0, \boldsymbol{\epsilon}_1), \quad \epsilon_1^\nu = (0, \boldsymbol{\epsilon}_2) \qquad \text{with} \qquad \boldsymbol{k}\cdot\boldsymbol{\epsilon}_1 = 0, \quad \boldsymbol{k}\cdot\boldsymbol{\epsilon}_2 = 0. \tag{3.10}$$

Further, we see that the $(k_1^\alpha - k_2^\alpha)$ should be expressed as

$$k_1^\alpha - k_2^\alpha = (0, 2\boldsymbol{k}). \tag{3.11}$$

Therefore, we can easily prove by now that the T-matrix should be exactly zero

$$T_{Z^0\to 2\gamma} = 0$$

which is due to the anti-symmetric nature of $\varepsilon_{\mu\nu\rho\alpha}$ where the non-zero part of the T-matrix should always satisfy the condition that μ, ν, ρ, α should be different from each other. This zero decay rate is known as the Landau-Yang theorem [5, 6]. Here, we should make a comment on the branching ratio of $\Gamma_{Z^0\to 2\gamma}/\Gamma$, and the present experimental upper limit shows [11]

$$(\Gamma_{Z^0\to 2\gamma}/\Gamma)_{exp} < 5.2 \times 10^{-5} \tag{3.12}$$

which is consistent with zero decay rate. Therefore, the theoretical prediction of the branching ratio is indeed consistent with experiments.

4. Group Theoretical Evaluation

At this point, we should make it clear that the spin states of two photons should be uniquely determined in terms of group theory terminology. First, we note that the polarization vector of photon state corresponds to a rank-one tensor $\boldsymbol{\epsilon}$, even though it is not necessarily the same as the angular momentum $\boldsymbol{L}$. From these two rank-one tensors ($\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$), one can construct rank-0 tensor (scalar)

$$[0] = \boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2 = \boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_1 \tag{4.1}$$

and rank-1 tensor (vector)

$$[1] = \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2 = -\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_1 \tag{4.2}$$

as well as the rank-2 tensor. In this case, one can easily prove that the rank-0 tensor (scalar) should be symmetric between two photon polarization vectors as can be easily seen from eq.(4.1). On the other hand, the rank-1 tensor (vector) should be anti-symmetric under the exchange of $1 \leftrightarrow 2$. Therefore, if one adds the two Feynman diagrams together, then they should vanish to zero, which is just the Landau-Yang theorem.

5. Conclusions

We have presented a new calculation of the weak vector boson of Z^0 decay into two photons. The calculated T-matrix does not have either linear nor logarithmic divergences, and the finite term is shown to vanish to zero due to the symmetry nature of the two photon states. This is consistent with the Landau-Yang theorem, and any physicists with sufficient skills of Feynman diagram evaluations should be able to check the result without any difficulties. Also, we confirm that the decay rate of the $\pi^0 \to 2\gamma$ process is reproduced well by Nishijima's calculation. Therefore, we see that all of the triangle diagrams do not have any divergences at all, and all of the observed decay rates are reproduced properly by the calculation. This is all what we can do in physics in connection with the triangle diagrams.

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