Chapter 9

Quantum Gravity

The quantum field theory of gravitation is constructed in terms of Lagrangian density of Dirac fields which couple to the electromagnetic field A_μ as well as the gravitational field $\mathcal{G}$. The gravity appears in the mass term as $m(1 + g\mathcal{G})\bar{\psi}\psi$ with the coupling constant of g. In addition to the gravitational force between fermions, the electromagnetic field A_μ interacts with the gravity as the fourth order effects and its strength amounts to α times the gravitational force. Therefore, the interaction of photon with gravity is not originated from Einstein's general relativity. Further, we present a renormalization scheme for the gravity and show that the graviton stays massless.

9.1 Problems of General Relativity

The motion of the earth is governed by the gravitational force between the earth and the sun, and the Newton equation is written as

$$m\ddot{\boldsymbol{r}} = -G_0 mM \,\frac{\boldsymbol{r}}{r^3}\,, \tag{9.1}$$

where G_0, m and M denote the gravitational constant, the mass of the earth and the mass of the sun, respectively. This is the classical mechanics which works quite well.

The gravitational potential that appears in eq.(9.1) is experimentally determined. However, the theoretical derivation of the gravity cannot be achieved in any of the equations such as Newton equation or Maxwell equations. Einstein presented the equation of general relativity which should be some analogous equations to the Maxwell equations in the sense that the gravitational field should be determined by the equation of general relativity. However, since he employed the principle of equivalence which has nothing to do with real experiments, the general relativity became an equation that determines the metric tensor. This does not mean that one can determine the gravitational interaction, and indeed, we have completely lost the correct direction in physics. Therefore, we should find a theoretical frame work to determine the gravitational interaction with fermions in some way or the other.

Before constructing a theory that can describe the field equation under the gravity, we discuss the fundamental problems in the theory of general relativity. Basically, there are two serious problems in the general relativity, the lack of field equation under the gravity and the assumption of the principle of equivalence.

9.1.1 Field Equation of Gravity

When one wishes to write the Dirac equation for a particle under the gravitational interaction, then one faces to the difficulty. Since the Dirac equation for a hydrogen-like atom can be written as

$$\left(-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+m\beta-\frac{Ze^2}{r}\right)\Psi=E\Psi \tag{9.2a}$$

one may write the Dirac equation for the gravitational potential $V(r)=-\frac{G_0mM}{r}$ as

$$\left(-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+m\beta-\frac{G_0mM}{r}\right)\Psi=E\Psi. \tag{9.2b}$$

But there is no foundation for this equation. At least, one cannot write the Lagrangian density which can describe the Dirac equation for the gravitational interaction. This is clear since one does not know whether the interaction can be put into the zero-th component of a vector type or a simple scalar type in the Dirac equation. That is, it may be of the following type

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+\left(m-\frac{G_0mM}{r}\right)\beta\right]\Psi=E\Psi. \tag{9.2c}$$

In fact, this is a right Dirac equation for a particle in the gravitational potential.

9.1.2 Principle of Equivalence

The theory of general relativity is entirely based on the principle of equivalence. Namely, Einstein started from the Gedanken experiment that physics of the two systems (a system under the uniform external gravity and a system that moves with a constant acceleration) must be the same. This looks plausible from the experience on the earth. However, one can easily convince oneself that the system that moves with a constant acceleration cannot be defined properly since there is no such an isolated system in a physical world. The basic problem is that the assumption of the principle of equivalence is concerned with the two systems which specify space and time, not just the numbers in connection with the acceleration of a particle. Note that the acceleration of a particle is indeed connected to the gravitational acceleration, $\ddot{z}=-g$, but this is, of course, just the Newton equation. Therefore, the principle of equivalence inevitably leads Einstein to the space deformation. It is clear that physics must be the same between two inertia systems, and any assumption which contradicts this basic principle cannot be justified at all.

Frame or Coordinate Transformation

Besides, this problem can be viewed differently in terms of Lagrangian. For the system under the uniform external gravity, one can write the corresponding Lagrangian. On the other hand, there is no way to construct any Lagrangian for the system that moves with a constant acceleration. One can define a Lagrangian for a particle that moves with a constant acceleration, but one cannot write the system (or space and time) that moves with a constant acceleration.

Physics in one inertia frame must be equivalent to that of another inertia frame, and this requirement is very severe. It is not only a coordinate change of space and time with Lorentz transformation, but also physical observables must be the same between two systems. In this respect, the principle of equivalence violates this important condition, and therefore, it is very hard to accept the assumption of the principle of equivalence even with the most modest physical intuition.

9.1.3 General Relativity

Einstein generalized the Poisson type equation for gravity

$$\boldsymbol{\nabla}^2 \phi_g = 4\pi G_0 \rho$$

to the tensor equations which should have some similarities with the Maxwell equations. Therefore, he had to find some tensor quantity like the field strength $F_{\mu\nu}$ of the electromagnetic field, and the metric tensor $g_{\mu\nu}$ is chosen as the basic tensor field since he started from the principle of equivalence. Thus, the general relativity is the equation for the metric tensor $g_{\mu\nu}$ which, he believed, should be connected to the gravitational field ϕ_g. By noting that

$$g_{00} \simeq 1 + \frac{2\phi_g}{c^2}$$

together with

$$T_{00} \simeq \rho$$

with the energy momentum tensor of $T_{\mu\nu}$, he arrived at the equation of the general relativity

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G_0 T^{\mu\nu},$$

where $R^{\mu\nu}$ denotes the Ricci tensor which can be described in terms of the metric tensor $g_{\mu\nu}$. However, the physical meaning of the $g_{\mu\nu}$ is unclear, and that is the basic problem of the general relativity.

Mathematics vs Physics

It should be important to note that Einstein's equation is mathematically complicated, but physically it is just simple. First, we should understand the physics of the Poisson type equation, and the equation is to determine the gravitational field ϕ_g when there is a matter field density ρ. Just in the same way, Einstein's equation can describe the behavior of the metric tensor $g_{\mu\nu}$ when there is the energy-momentum tensor $T_{\mu\nu}$ which should be generated by the matter field density ρ. However, the Poisson type equation is unfortunately insufficient to determine the gravitational interaction with the matter field since one has to know in which way the matter field should be influenced by the gravitational field, and this should be determined by the equation for the matter field like the Dirac equation. In the same way, Einstein's equation is insufficient to find the interaction with the matter field. In addition, the energy-momentum tensor $T_{\mu\nu}$ cannot be defined well unless one has a good field theoretical picture of fermions. In this respect, it is most important to find the Lagrangian density which includes the gravitational field interacting with the fermions, and this will be explained in the next section.

Before going to the discussion of the Lagrangian density of the gravity, it should be important to clarify the origin of the coordinate x_μ in the metric tensor $g_{\mu\nu}(x)$ from where it is measured. From Einstein's equation, it is clear that the origin of the coordinate should be found in the matter field center. Therefore, one can see that the metric tensor $g_{\mu\nu}$ should be in contradiction with the principle of special relativity since its space and time become different from the space and time of the other inertia system. This peculiar behavior of the metric tensor $g_{\mu\nu}$ is just the result of the principle of equivalence which is not consistent with the principle of special relativity as discussed above.

9.2 Lagrangian Density for Gravity

We should start from constructing the quantum mechanics of the gravitation. In other words, we should find the Dirac equation for electron when it moves in the gravitational potential. In this chapter, we present a model Lagrangian density which can describe electrons interacting with the electromagnetic field A_μ as well as the gravitational field $\mathcal{G}$.

9.2.1 Lagrangian Density for QED

We first write the Lagrangian density for electrons interacting with the electromagnetic field A_μ as given in eq.(5.1)

$$\mathcal{L}_{el} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{9.3}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

This Lagrangian density of QED is best studied and is most reliable in many respects. In particular, the renormalization scheme of QED is theoretically well understood and is experimentally well examined, and there is no problem at all in the perturbative treatment of QED. All the physical observables can be described in terms of the free Fock space terminology after the renormalization, and therefore one can compare any prediction of the physical quantities with experiment. However, it should be noted that QED is the only field theory model in four dimensions which works perfectly well without any conceptual difficulties.

9.2.2 Lagrangian Density for QED plus Gravity

Now, we propose to write the Lagrangian density for electrons interacting with the electromagnetic field as well as the gravitational field $\mathcal{G}$ [36]

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - e\bar{\psi}\gamma^{\mu}A_{\mu}\psi - m(1+g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_{\mu}\mathcal{G}\partial^{\mu}\mathcal{G}, \tag{9.4}$$

where the gravitational field $\mathcal{G}$ is assumed to be a massless scalar field. It is easy to prove that the new Lagrangian density is invariant under the local gauge transformation

$$A_{\mu} \to A_{\mu} + \partial_{\mu}\chi, \quad \psi \to e^{-ie\chi}\psi. \tag{9.5}$$

This is, of course, quite important since the introduction of the gravitational field does not change the most important local symmetry.

9.2.3 Dirac Equation with Gravitational Interactions

Now, one can easily obtain the Dirac equation for electrons from the new Lagrangian density

$$i\gamma^{\mu}\partial_{\mu}\psi - e\gamma^{\mu}A_{\mu}\psi - m(1+g\mathcal{G})\psi = 0. \tag{9.6}$$

Also, one can write the equation of motion of gravitational field

$$\partial_{\mu}\partial^{\mu}\mathcal{G} = -mg\bar{\psi}\psi. \tag{9.7}$$

The symmetry property of the new Lagrangian density can be easily examined, and one can confirm that it has a right symmetry property under the time reversal transformation, parity transformation and the charge conjugation.

9.2.4 Total Hamiltonian for QED plus Gravity

The Hamiltonian can be constructed from the Lagrangian density in eq.(9.4)

$$H = \int \left\{ \bar{\psi}\big(-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + m(1+g\mathcal{G})\big)\psi - e\boldsymbol{j}\cdot\boldsymbol{A} \right\} d^3r + \frac{e^2}{8\pi}\int \frac{j_0(\boldsymbol{r}')j_0(\boldsymbol{r})\, d^3r\, d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}$$
$$+\frac{1}{2}\int \left(\dot{\boldsymbol{A}}^2 + (\boldsymbol{\nabla}\times\boldsymbol{A})^2 \right) d^3r + \frac{1}{2}\int \left(\dot{\mathcal{G}}^2 + (\boldsymbol{\nabla}\mathcal{G})^2 \right) d^3r, \tag{9.8}$$

where j_{μ} is defined as $j_{\mu} = \bar{\psi}\gamma_{\mu}\psi$. In this expression of the Hamiltonian, the gravitational energy is still written without making use of the equation of motion. In the next section, we will treat the gravitational energy and rewrite it into an expression which should enable us to easily understand the structure of gravitational force between fermions.

9.3 Static-dominance Ansatz for Gravity

In eq.(9.4), the gravitational field $\mathcal{G}$ is introduced as a *real scalar* field, and therefore it cannot be a physical observable as a classical field [48, 80]. In this case, since the real part of the right hand side in eq.(9.7) should be mostly time independent, it may be reasonable to assume that the gravitational field $\mathcal{G}$ can be written as the sum of the static and time-dependent terms and that the static part should carry the information of diagonal term in the external source term. Thus, the gravitational field $\mathcal{G}$ is assumed to be written as

$$\mathcal{G} = \mathcal{G}_0(\boldsymbol{r}) + \bar{\mathcal{G}}(x), \tag{9.9}$$

where $\mathcal{G}_0(\boldsymbol{r})$ does not depend on time. This ansatz is only a sufficient condition, and its validity cannot be verified mathematically, but it can be examined experimentally.

The equations of motion for $\mathcal{G}_0(\boldsymbol{r})$ and $\bar{\mathcal{G}}(x)$ become

$$\boldsymbol{\nabla}^2 \mathcal{G}_0 = mg\rho_g, \tag{9.10}$$

$$\partial_\mu \partial^\mu \bar{\mathcal{G}}(x) = -mg\Big\{ (\bar{\psi}\psi)_{[\mathrm{non-diagonal}]} + (\bar{\psi}\psi)_{[\mathrm{diagonal\ rest}]} \Big\}, \tag{9.11}$$

where ρ_g is defined as

$$\rho_g \equiv (\bar{\psi}\psi)_{[\mathrm{diagonal}]}, \tag{9.12}$$

where $(\bar{\psi}\psi)_{[\mathrm{diagonal}]}$ denotes the diagonal part of the $\bar{\psi}\psi$, that is, the terms proportional to $[a_{\boldsymbol{k}}^{\dagger(s)} a_{\boldsymbol{k}'}^{(s)} - b_{\boldsymbol{k}}^{\dagger(s)} b_{\boldsymbol{k}'}^{(s)}]$ of the fermion operators which will be defined in eq.(9.19). Further, $(\bar{\psi}\psi)_{[\mathrm{non-diagonal}]}$ term is a non-diagonal part which is connected to the creation and annihilation of fermion pairs, that is, $[a_{\boldsymbol{k}}^{\dagger(s)} b_{-\boldsymbol{k}'}^{\dagger(s)} + b_{-\boldsymbol{k}'}^{(s)} a_{\boldsymbol{k}}^{(s)}]$ of the fermion operators. In addition, the term $(\bar{\psi}\psi)_{[\mathrm{diagonal\ rest}]}$ denotes time dependent parts of the diagonal term in the fermion density, and this may also have some effects when the gravity is quantized.

In this case, we can solve eq.(9.10) exactly and find a solution

$$\mathcal{G}_0(\boldsymbol{r}) = -\frac{mg}{4\pi} \int \frac{\rho_g(\boldsymbol{r}')}{|\boldsymbol{r}' - \boldsymbol{r}|}\, d^3r', \tag{9.13}$$

which is a special solution that satisfies eq.(9.7), but not the general solution. Clearly as long as the solution can satisfy the equation of motion of eq.(9.7), it is physically sufficient. The solution of eq.(9.13) is quite important for the gravitational interaction since this is practically a dominant gravitational force in nature.

Here, we assume that the diagonal term of $(\bar{\psi}\psi)_{[\mathrm{diagonal}]}$ is mostly time independent, and in this case, the static gravitational energy which we call H_G^S can be written as

$$H_G^S = mg \int \rho_g \mathcal{G}_0\, d^3r + \frac{1}{2} \int (\boldsymbol{\nabla}\mathcal{G}_0)^2\, d^3r = -\frac{m^2 G_0}{2} \int \frac{\rho_g(\boldsymbol{r}')\rho_g(\boldsymbol{r})}{|\boldsymbol{r}' - \boldsymbol{r}|}\, d^3r\, d^3r', \tag{9.14}$$

where the gravitational constant G_0 is related to the coupling constant g as

$$G_0 = \frac{g^2}{4\pi}\,. \tag{9.15}$$

Eq.(9.14) is just the gravitational interaction energy for the matter fields, and one sees that the gravitational interaction between electrons is always attractive. This is clear since the gravitational field is assumed to be a massless scalar. It may also be important to note that the H_G^S of eq.(9.14) is obtained without making use of the perturbation theory, and it is indeed exact, apart from the static ansatz of the field $\mathcal{G}_0(\boldsymbol{r})$.

9.4 Quantization of Gravitational Field

In quantum field theory, we should quantize fields. For fermion fields, we should quantize the Dirac field by the anti-commutation relations of fermion operators. This is required from the experiment in terms of the Pauli principle, that is, a fermion can occupy only one quantum state. In order to accommodate this experimental fact, we should always quantize the fermion fields with the anti-commutation relations. On the other hand, for gauge fields, we must quantize the vector field in terms of the commutation relation which is also required from the experimental observation that one photon is emitted by the transition between $2p$-state and $1s$-state in hydrogen atoms. That is, a photon is created from the vacuum of the electromagnetic field, and therefore the field quantization is an absolutely necessary procedure. However, it is not very clear whether the gravitational field $\mathcal{G}$ should be quantized according to the bosonic commutation relation or not. In fact, there must be two choices concerning the quantization of the gravitational field $\mathcal{G}$.

9.4.1 No Quantization of Gravitational Field

As the first choice, we may take a standpoint that the gravitational field $\mathcal{G}$ should not be quantized since there is no requirement from experiments. In this sense, there is no definite reason that we have to quantize the scalar field and therefore the gravitational field $\mathcal{G}$ should remain to be a classical field. In this case, we do not have to worry about the renormalization of the graviton propagator, and we obtain the gravitational interaction between fermions as we saw it in eq.(9.14) which is always attractive, and this is consistent with the experimental requirement.

9.4.2 Quantization Procedure

Now, we take the second choice and should quantize the gravitational field $\bar{\mathcal{G}}$. This can be done just in the same way as usual scalar fields

$$\bar{\mathcal{G}}(x) = \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2V\omega_k}} \left[d_k e^{-i\omega_k t + i\boldsymbol{k}\cdot\boldsymbol{r}} + d_k^\dagger e^{i\omega_k t - i\boldsymbol{k}\cdot\boldsymbol{r}} \right], \tag{9.16}$$

where $\omega_k = |\boldsymbol{k}|$. The annihilation and creation operators d_k and $d_k^\dagger$ are assumed to satisfy the following commutation relations

$$\left[d_{\boldsymbol{k}}, d_{\boldsymbol{k}'}^\dagger \right] = \delta_{\boldsymbol{k},\boldsymbol{k}'} \tag{9.17}$$

and all other commutation relations should vanish. Since the graviton can couple to the time dependent external field which is connected to the creation or annihilation of the fermion pairs, the graviton propagator should be affected from the vacuum polarization of fermions. Therefore, we should carry out the renormalization procedure of the graviton propagator such that it can stay massless. We will discuss the renormalization procedure in the later section.

9.4.3 Graviton

Once the gravitational field $\mathcal{G}$ is quantized, then the graviton should appear. From eq.(9.16), one can see that the graviton can indeed propagate as a free massless particle after it is quantized, and this situation is just the same as the gauge field case in QED, namely, photon after the quantization becomes a physical observable. However, it should be noted that the gauge field has a special feature in the sense that the classical gauge field ($\boldsymbol{A}$) is gauge dependent and therefore it is not a physical observable. After the gauge fixing, the gauge field can be quantized since one can uniquely determine the gauge field from the equation of motion, and therefore its quantization is possible.

On the other hand, the gravitational field is assumed to be a real scalar field, and therefore it cannot be a physical observable as a classical field [48, 80]. Only after the quantization, it becomes a physical observable as a graviton, and this can be seen from eq.(9.16) since the creation of the graviton should be made through the second term of eq.(9.16). In this case, the graviton field is a complex field which is an eigenstate of the momentum and thus it is a free graviton state, which can propagate as a free particle.

9.5 Interaction of Photon with Gravity

From the Lagrangian density of eq.(9.4), one sees that photon should interact with the gravity in the fourth order Feynman diagrams as shown in Fig.9.1. The interaction Hamiltonian H_I can be written as

$$H_I = \int \left(mg\mathcal{G}\bar{\psi}\psi - e\bar{\psi}\boldsymbol{\gamma}\psi \cdot \boldsymbol{A} \right) d^3r, \tag{9.18}$$

where the fermion field ψ is quantized in the normal way

$$\psi(\boldsymbol{r},t) = \sum_{\boldsymbol{p},s} \frac{1}{\sqrt{L^3}} \left(a_{\boldsymbol{p}}^{(s)} u_{\boldsymbol{p}}^{(s)} e^{i\boldsymbol{p}\cdot\boldsymbol{r} - iE_{\boldsymbol{p}}t} + b_{\boldsymbol{p}}^{\dagger(s)} v_{\boldsymbol{p}}^{(s)} e^{-i\boldsymbol{p}\cdot\boldsymbol{r} + iE_{\boldsymbol{p}}t} \right), \tag{9.19}$$

where $u_{\boldsymbol{p}}^{(s)}$ and $v_{\boldsymbol{p}}^{(s)}$ denote the spinor part of the plane wave solutions of the free Dirac equation. $a_{\boldsymbol{p}}^{(s)}$ and $b_{\boldsymbol{p}}^{(s)}$ are annihilation operators for particle and anti-particle states, and they should satisfy the following anti-commutation relations,

$$\{a_{\boldsymbol{p}}^{(s)}, a^{\dagger}{}_{\boldsymbol{p}'}^{(s')}\} = \delta_{s,s'}\delta_{\boldsymbol{p},\boldsymbol{p}'}, \quad \{b_{\boldsymbol{p}}^{(s)}, b^{\dagger}{}_{\boldsymbol{p}'}^{(s')}\} = \delta_{s,s'}\delta_{\boldsymbol{p},\boldsymbol{p}'} \tag{9.20}$$

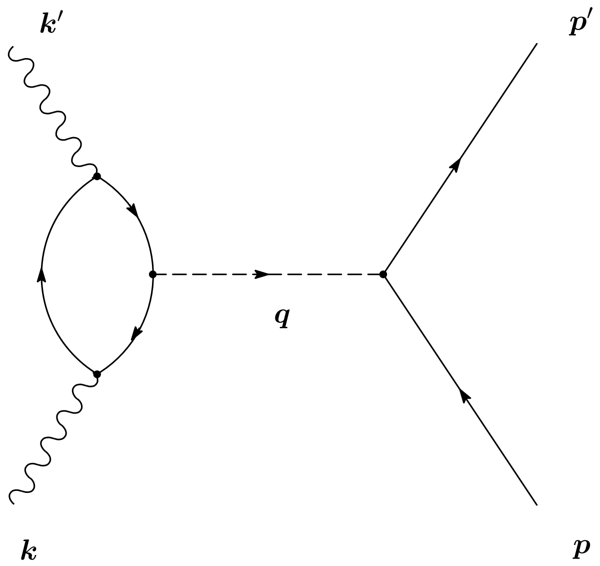

Fig. 9.1: The fourth order Feynman diagram

and all other anticommutation relations should vanish. The gauge field $\boldsymbol{A}$ can be quantized as given in eq.(3.24) in Chapter 3

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k}} \sum_{\lambda=1}^{2} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}} \boldsymbol{\epsilon}^{\lambda}(\boldsymbol{k}) \left[c_{\boldsymbol{k},\lambda} e^{-ikx} + c_{\boldsymbol{k},\lambda}^{\dagger} e^{ikx} \right], \tag{9.21}$$

where $\omega_{\boldsymbol{k}} = |\boldsymbol{k}|$. The polarization vector $\boldsymbol{\epsilon}^{\lambda}(\boldsymbol{k})$ should satisfy the following relations

$$\boldsymbol{\epsilon}^{\lambda}(\boldsymbol{k}) \cdot \boldsymbol{k} = 0, \quad \boldsymbol{\epsilon}^{\lambda}(\boldsymbol{k}) \cdot \boldsymbol{\epsilon}^{\lambda'}(\boldsymbol{k}) = \delta_{\lambda,\lambda'}. \tag{9.22}$$

The annihilation and creation operators $c_{\boldsymbol{k},\lambda}$, $c_{\boldsymbol{k},\lambda}^{\dagger}$ should satisfy the following commutation relations

$$[c_{\boldsymbol{k},\lambda}, c_{\boldsymbol{k}',\lambda'}^{\dagger}] = \delta_{\boldsymbol{k},\boldsymbol{k}'} \delta_{\lambda,\lambda'} \tag{9.23}$$

and all other commutation relations should vanish.

The calculation of the S-matrix can be carried out in a straightforward way [14, 89, 94], and we can write

$$S = (ie)^2 \epsilon_{\mu}^{\lambda}(k) \epsilon_{\nu}^{\lambda'}(k') \left(\frac{mm'g^2}{q^2} \right) \bar{u}(p') u(p)$$

$$\times \int \frac{d^4a}{(2\pi)^4} \mathrm{Tr} \left[\gamma_{\mu} \frac{i}{d - m + i\epsilon} \gamma_{\nu} \frac{i}{\not{b} - m + i\epsilon} \frac{i}{\not{c} - m + i\epsilon} \right], \tag{9.24}$$

where k and k' denote the four momenta of the initial and final photons while p and p' denote the four momenta of the initial and final fermions, respectively. m and m' denote the mass of the fermion for the vacuum polarization and the mass of the external fermion. a, b, c and q can be written in terms of k and p as

$$q = p' - p, \quad k = a - b, \quad k' = a - c, \quad q = k - k'.$$

Therefore, the S-matrix can be written as

$$S = ie^2 mm' g^2 \epsilon_\mu^\lambda(k)\epsilon_\nu^{\lambda'}(k')$$
$$\times \frac{1}{q^2}\,\bar{u}(p')u(p)\int \frac{d^4a}{(2\pi)^4}\,\frac{1}{a^2-m^2}\,\frac{1}{(a-k)^2-m^2}\,\frac{1}{(a-k')^2-m^2}$$
$$\times \mathrm{Tr}\left[\gamma_\mu(\not{a}+m)\gamma_\nu(\not{a}-\not{k}+m)(\not{a}-\not{k}'+m)\right]. \tag{9.25}$$

Since the term proportional to q does not contribute to the interaction, we can safely approximate in the evaluation of the trace and the integration of a as

$$k' \approx k.$$

Now, we define the trace part as

$$N_{\mu\nu} = \mathrm{Tr}\Big[\gamma_\mu(\not{a}+m)\gamma_\nu(\not{a}-\not{k}+m)(\not{a}-\not{k}'+m)\Big], \tag{9.26a}$$

which can be evaluated as

$$N_{\mu\nu} = 4m\Big[(k^2-a^2+m^2)g_{\mu\nu} + 4a_\mu a_\nu - 2a_\mu k_\nu - 2a_\nu k_\mu\Big]. \tag{9.26b}$$

Defining the integral by

$$I_{\mu\nu} \equiv \int \frac{d^4a}{(2\pi)^4}\,\frac{N_{\mu\nu}}{(a^2-m^2)[(a-k)^2-m^2]\,[(a-k')^2-m^2]} \tag{9.27}$$

we can rewrite it using Feynman integral

$$I_{\mu\nu} = 2\int \frac{d^4a}{(2\pi)^4}\int_0^1 z\,dz\,\frac{N_{\mu\nu}}{[(a-kz)^2-m^2+z(1-z)k^2]^3}\,. \tag{9.28}$$

Therefore, introducing the variable $w = a - kz$ we obtain the S-matrix as

$$S = 8ie^2m^2m'g^2\epsilon_\mu^\lambda(k)\epsilon_\nu^{\lambda'}(k')\,\frac{1}{q^2}\,\bar{u}(p')u(p)\int_0^1 z\,dz$$
$$\times\int\frac{d^4w}{(2\pi)^4}\left[\frac{(-w^2g_{\mu\nu}+4w_\mu w_\nu)}{[w^2-m^2+z(1-z)k^2]^3} + \frac{\left\{m^2+k^2(1-z^2)\right\}g_{\mu\nu}-4k_\mu k_\nu z(1-z)}{[w^2-m^2+z(1-z)k^2]^3}\right]. \tag{9.29}$$

The first part of the integration can be carried out in a straightforward way and we find

$$\int \frac{d^4w}{(2\pi)^4} \frac{(-w^2 g_{\mu\nu} + 4w_\mu w_\nu)}{[w^2 - m^2 + z(1-z)k^2]^3} = 0.$$

Thus, the two divergent parts just cancel with each other, and the cancellation here is not due to the regularization as employed in the self-energy diagrams in QED, but it is a kinematical and thus rigorous result. This situation is quite similar to the Feynman diagram of $\pi^0 \to 2\gamma$ decay process [94] and the calculated result of the Feynman diagram is indeed finite and is consistent with the experiment.

The finite part can be easily evaluated [36], and therefore we obtain the S-matrix as

$$S = \frac{e^2}{8\pi^2} m^2 m' g^2 (\epsilon^\lambda \epsilon^{\lambda'}) \frac{1}{q^2} \bar{u}(p')u(p), \tag{9.30}$$

where we made use of the relation $k^2 = 0$ for free photon at the end of the calculation.

9.6 Renormalization Scheme for Gravity

At the present stage, it is difficult to judge whether we should quantize the gravitational field or not. At least, there is no experiment which shows any necessity of the quantization of the gravity. Nevertheless, it should be worth checking whether the gravitational interaction with fermions can be renormalizable or not. We know that the interaction of the gravity with fermions is extremely small, but we need to examine whether the graviton can stay massless or not within the perturbation scheme.

Here, we present a renormalization scheme for the scalar field theory which couples to fermion fields. The renormalization scheme for scalar fields is formulated just in the same way as the QED scheme since QED is most successful.

9.6.1 Self-Energy of Graviton

First, we discuss the self-energy of graviton in gravitational interaction. As shown in Appendix J, the self-energy diagram of photon should not be considered for the renormalization procedure in QED. In the same manner, we see that there is no renormalization procedure necessary for the self-energy of graviton. Intuitively, this can be easily understood from eq.(9.7)

$$\partial_\mu \partial^\mu \mathcal{G} = -mg\bar{\psi}\psi. \tag{9.7}$$

As can be seen, the gravitational field $\mathcal{G}$ does not appear in the right hand side of eq.(9.7). This means that the change of the gravitational field from the second order perturbative calculations should be described by the fermion fields and cannot be written in terms of the gravitational field $\mathcal{G}$.

Therefore, the gravitational interaction is not affected from the graviton self-energy diagram. In the S-matrix evaluation, one sometimes finds that the calculated Feynman diagrams do not have any corresponding physical processes and the self-energy diagram of the graviton is just the case. Thus the graviton stays always massless.

Even though there is no practically interesting observables in higher order perturbation diagrams in the gravitational interaction, it is quite nice and transparent that the graviton propagator is not affected by the perturbation theory.

9.6.2 Fermion Self-Energy from Gravity

The fermion self-energy term in QED is calculated to be

$$\begin{aligned}\Sigma_{QED}(p) &= -ie^2 \int \frac{d^4k}{(2\pi)^4}\, \gamma_\mu \frac{1}{\not{p} - \not{k} - m} \gamma^\mu \frac{1}{k^2} \\ &= \frac{e^2}{8\pi^2} \ln\left(\frac{\Lambda}{m}\right)(-\not{p} + 4m) + \text{finite terms.}\end{aligned} \tag{9.31}$$

In the same way, we can calculate the fermion self-energy due to the gravity

$$\begin{aligned}\Sigma_G(p) &= im^2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{p} - \not{k} - m} \frac{1}{k^2} \\ &= -\frac{m^2g^2}{8\pi^2} \ln\left(\frac{\Lambda}{m}\right)(-\not{p} + 4m) + \text{finite terms}\,,\end{aligned} \tag{9.32}$$

which is just the same as the QED case, apart from the factor in front. Therefore, the renormalization procedure can be carried out just in the same way as the QED case since the total fermion self-energy term within the present model becomes

$$\Sigma(p) = \frac{1}{8\pi^2} \ln\left(\frac{\Lambda}{m}\right)(e^2 - m^2g^2)(-\not{p} + 4m) + \text{finite terms.} \tag{9.33}$$

9.6.3 Vertex Correction from Gravity

Concerning the vertex corrections which arise from the gravitational interaction and electromagnetic interaction with fermions, it may well be that the vertex corrections do not become physically very important. It is obviously too small to measure any effects of the higher order terms from the gravity and electromagnetic interactions. However, we should examine the renormalizability of the vertex corrections and can show that they are indeed well renormalized into the wave function. The vertex corrections from the electromagnetic interaction and the gravity can be evaluated as

$$\Lambda_{QED}(k,q) = imge^2 \int \frac{d^4p}{(2\pi)^4} \left[\gamma_\mu \frac{1}{(\not{k} - \not{p} - m)(\not{k} - \not{p} - \not{q} - m)p^2} \gamma^\mu\right], \tag{9.34a}$$

$$\Lambda_G(k,q) = -im^3g^3 \int \frac{d^4p}{(2\pi)^4} \left[\frac{1}{(\not{k} - \not{p} - m)(\not{k} - \not{p} - \not{q} - m)p^2}\right]. \tag{9.34b}$$

We can easily calculate the integrations and obtain the total vertex corrections for the zero momentum case of $q = 0$ as

$$\Lambda(k,0) = \Lambda_{QED}(k,0) + \Lambda_G(k,0) = \frac{mg}{\pi^2} \ln\left(\frac{\Lambda}{m}\right)(e^2 - m^2g^2) + \text{finite terms}\,, \quad (9.35)$$

which is logarithmic divergence and is indeed renormalizable just in the same way as the QED case.

9.6.4 Renormalization Procedure

Since the infinite contributions to the fermion self-energy and to the vertex corrections in the second order diagrams are just the same as the QED case, one can carry out the renormalization procedure just in the same way as the QED case. In this way, we can achieve a successful renormalization scheme for the gravity, even though we do not know any occasions in which the higher order contributions may become physically important.

9.7 Gravitational Interaction of Photon with Matter

From eq.(9.30), one finds that the gravitational potential $V(r)$ for photon with matter field can be written as

$$V(r) = -\frac{G_0 \alpha m_t^2 M}{2\pi}\,\frac{1}{r}\,, \quad (9.36)$$

where m_t and M denote the sum of all the fermion masses and the mass of matter field, respectively. α denotes the fine structure constant $\alpha = \frac{1}{137}$. In this case, the equation of motion for photon $\boldsymbol{A}_\lambda$ under the gravitational field becomes

$$\left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 - \frac{G_0 \alpha m_t^2 M}{2\pi}\,\frac{1}{r}\right)\boldsymbol{A}_\lambda = 0. \quad (9.37)$$

Assuming the time dependence of the photon field $\boldsymbol{A}_\lambda$ as

$$\boldsymbol{A}_\lambda = \boldsymbol{\epsilon}_\lambda e^{-i\omega t} A_0(\boldsymbol{r}) \quad (9.38)$$

we obtain

$$\left(-\boldsymbol{\nabla}^2 - \frac{G_0 \alpha m_t^2 M}{2\pi}\,\frac{1}{r}\right)A_0(\boldsymbol{r}) = \omega^2 A_0(\boldsymbol{r}). \quad (9.39)$$

This equation shows that there is no bound state for photon even for the strong coupling limit of $G_0 \to \infty$.

9.7.1 Photon-Gravity Scattering Process

Eq.(9.39) can be written with $|\boldsymbol{k}| = \omega$ as

$$\left(\boldsymbol{\nabla}^2 + \boldsymbol{k}^2\right) A_0(\boldsymbol{r}) = -\frac{G_0 \alpha m_t^2 M}{2\pi} \frac{1}{r} A_0(\boldsymbol{r}), \tag{9.40}$$

which is just the same as the scattering process of a particle under the Coulomb interaction. When the non-relativistic particle with its mass m_0 and momentum $\boldsymbol{k}$ scatters elastically with a point nucleus with its charge Z, the Schrödinger equation becomes

$$\left(\boldsymbol{\nabla}^2 + \boldsymbol{k}^2\right) \psi(\boldsymbol{r}) = -\frac{2m_0 Z e^2}{r} \psi(\boldsymbol{r}). \tag{9.41}$$

The solution of eq.(9.41) is well studied and therefore we can make use of this equation to solve eq.(9.39). In this case, we can obtain the differential cross section of the photon-gravity scattering process

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_g^2}{16\omega^4 \sin^4 \frac{\theta}{2}}\,, \tag{9.42}$$

where α_g is defined as

$$\alpha_g = \frac{G_0 \alpha m_t^2 M}{2\pi}\,. \tag{9.43}$$

This differential cross section is just the same as the Rutherford cross section.

9.8 Cosmology

What should be a possible picture of our universe in the new quantum theory of gravity? By now we have sufficient knowledge concerning the cosmology how the present universe is created and what should be a fate of the present universe. Below is a simple picture one can easily draw, even though it is almost a story. In order to make it into physics, hard works may be required, though it must be a doable task.

9.8.1 Cosmic Fireball Formation

Since the gravity is always attractive, it is clear that all of the galaxies should eventually get together. A question may arise in which way these galaxies would collapse into a Cosmic Fireball. It is most likely true that, after the end of the expansion of the present universe, a few galaxies should coalesce into a larger galaxy, and this coalescence should take place repeatedly until two or three giant clusters of galaxies should be formed. Finally, these giant clusters would eventually collide into a Cosmic Fireball which should be quite similar to the initial stage of the big bang. After the Cosmic Fireball is created, it should rapidly expand, and during the expansion, light nuclei should be created. In this picture, galaxies should be naturally formed since the expansion after the explosion should not be very uniform. This is in contrast to the big bang cosmology in which the galaxy formation must be quite difficult since the big bang should be extremely uniform.

In this respect, the universe should repeatedly make the same formation of galaxies. The universe should have existed from the infinite time of past, and it should make the galaxy formation and collision in the infinite time of future.

Here, it should be noted that the concept of the infinite time of past or future is beyond the understanding of human being. Also, the whole universe should have infinite space, but again the infinite space should not be a target of physics research.

9.8.2 Relics of Preceding Universe

According to the present picture of the universe, there may well be some relics of the preceding universe before the Cosmic Fireball.

Large Scale Structure of Universe

In the present universe, there is a large scale structure of the universe among cluster of galaxies such as the Great Attractor. This should be related to the remnants of the Cosmic Fireball formation when the preceding universe got together into the Cosmic Fireball.

Photon Baryon Ratio

Another possible relic must be the large number of photons compared to the number of baryons in the present universe. This photon-baryon ratio may well be understood in terms of the relic of photons in the preceding universe since photon has some interactions with strong gravitational fields and therefore some of photons may be trapped during the Cosmic Fireball formation. On the other hand, neutrino should not be trapped due to the lack of the interactions with baryons. Therefore, the number of neutrinos must be much smaller than the number of photons in the present universe.

9.8.3 Remarks

The gravitational interaction appears always as the mass term and induces always the attractive force between fermions. In addition, there is an interaction between photon and the gravity as the fourth order Feynman diagrams. The behavior of photon under the gravitational field may have some similarity with the result of the general relativity.

The renormalization procedure of the gravitational interaction is carried out in the same way as the QED case, and therefore the propagator of the graviton stays massless, which is just the same as the QED case in which photon stays always massless. This can be easily understood from the observation that the self-energy of photon as well as graviton should not be considered for the renormalization scheme.

Here, it is still an open question whether the gravitational field should be quantized or not. This is basically because there is no definite requirement from experiment for the quantization. For the quantized theory of gravitational field, one may ask as to whether there is any method to observe a graviton or not. The graviton should be created through the fermion pair annihilation. Since this graviton can propagate as a free graviton like a photon, one may certainly have some chance to observe it through the creation of the fermion pair. But this probability must be extremely small since the coupling constant is very small, and there is no enhancement in this process unless a strong gravitational field like a neutron star may rapidly change as a function of time.

9.9 Time Shifts of Mercury and Earth Motions

The new gravity model is applied to the description of the observed advance shifts of the Mercury perihelion, the earth rotation and the GPS satellite motion. First, we obtain the gravitational potential which can be calculated from the non-relativistic reduction of the Dirac equation in terms of the Foldy-Wouthuysen transformation. Then, we should make the classical limit of the Hamiltonian so that we can obtain the classical potential for the gravity.

9.9.1 Non-relativistic Gravitational Potential

The Hamiltonian of the Dirac equation in the gravitational field can be written as

$$H = -i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + \left(m - \frac{GmM}{r}\right)\beta, \tag{9.44}$$

where M denotes the mass of the gravity center. This Hamiltonian can be easily reduced to the non-relativistic equation of motion by making use of the Foldy-Wouthuysen transformation [14]. Here, we only write the result in terms of the Hamiltonian H [46]

$$H = m + \frac{\boldsymbol{p}^2}{2m} - \frac{GmM}{r} + \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2 - \frac{1}{2m^2}\frac{GMm}{r^3}(\boldsymbol{s}\cdot\boldsymbol{L}), \tag{9.45}$$

where the last term denotes the spin-orbit force, but we do not consider it here. Now, we make the classical limit to derive the Newton equation. In this case, it is safe to assume the factorization ansatz for the third term, that is,

$$\left\langle \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2 \right\rangle = \left\langle \frac{1}{2m^2}\frac{GmM}{r} \right\rangle \left\langle \boldsymbol{p}^2 \right\rangle. \tag{9.46}$$

By making use of the Virial theorem for the gravitational potential

$$\left\langle \frac{\boldsymbol{p}^2}{m} \right\rangle = \left\langle \frac{GmM}{r} \right\rangle \tag{9.47}$$

we obtain the new gravitational potential for the Newton equation

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2, \tag{9.48}$$

where we explicitly write the light velocity c in the last term of the equation.

9.9.2 Time Shifts of Mercury, GPS Satellite and Earth

The Newton equation with the new gravitational potential can be written as

$$m\ddot{r} = -\frac{GmM}{r^2} + \frac{\ell^2}{mr^3} + \frac{G^2M^2m}{c^2r^3}. \tag{9.49}$$

Therefore, we can introduce a new angular momentum L as

$$L^2 \equiv \ell^2 + \frac{G^2M^2m^2}{c^2}. \tag{9.50}$$

Further, we define the angular velocity ω and radius R by

$$\omega \equiv \frac{\ell}{mR^2}, \quad R \equiv \frac{\ell^2}{GMm^2(1-\varepsilon^2)^{\frac{3}{4}}}, \tag{9.51a}$$

where ε denotes the eccentricity. Correspondingly, we can define a new angular velocity Ω associated with ω as

$$\Omega^2 \equiv \omega^2 + \frac{G^2M^2}{c^2R^4} = \omega^2(1+\eta), \tag{9.51b}$$

where η is defined as

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2}. \tag{9.52}$$

The equation (9.49) can be immediately solved, and one finds the solution of the orbit

$$r = \frac{A}{1+\varepsilon\cos\left(\frac{L}{\ell}\varphi\right)}, \tag{9.53}$$

where A and ε are given as

$$A = \frac{L^2}{GMm^2}, \quad \varepsilon = \sqrt{1 + \frac{2L^2E}{m(GmM)^2}}. \tag{9.54}$$

Physical observables can be obtained by integrating $\dot{\varphi} = \frac{\ell}{mr^2}$ over the period T

$$\frac{\ell}{m}\int_0^T dt = \int_0^{2\pi} r^2 d\varphi = A^2 \int_0^{2\pi} \frac{1}{\left(1+\varepsilon\cos\left(\frac{L}{\ell}\varphi\right)\right)^2} d\varphi. \tag{9.55}$$

This can be easily calculated to be

$$\omega T = 2\pi(1+2\eta)\,(1-\varepsilon\eta) \simeq 2\pi\{1+(2-\varepsilon)\eta\}, \tag{9.56}$$

where ε is assumed to be small. Therefore, the new gravity potential gives rise to the advance shift of the time shift, and it can be written as

$$\left(\frac{\Delta T}{T}\right)_{th} \simeq (2-\varepsilon)\eta. \tag{9.57}$$

This is a physical observable which indeed can be compared to experiment.

9.9.3 Mercury Perihelion Shift

The Mercury perihelion advance shift $\Delta\theta$ is well known to be [92]

$$\Delta\theta \simeq 43\,'' \ \text{per} \ 100\,\text{year}. \tag{9.58}$$

Since Mercury has the 0.24 year period, it can amount to the shift ratio $\delta\theta$

$$\delta\theta_{obs} \equiv \left(\frac{\Delta T}{T}\right)_{obs} \simeq 8.0 \times 10^{-8}. \tag{9.59}$$

The theoretical calculation of the new gravity model shows

$$\eta = \frac{G^2 M^2}{c^2 R^4 \omega^2} \simeq 2.65 \times 10^{-8} \tag{9.60}$$

where the following values are used for the Mercury case

$$R = 5.73 \times 10^{10}\ \text{m}, \quad M = 1.989 \times 10^{30}\ \text{kg}, \quad \omega = 8.30 \times 10^{-7}.$$

Therefore, the theoretical shift ratio $\delta\theta_{th}$ becomes

$$\delta\theta_{th} \equiv \left(\frac{\Delta T}{T}\right)_{th} \simeq 4.8 \times 10^{-8} \tag{9.61}$$

which should be compared to the observed value in eq.(9.59). As can be seen, this agreement is indeed remarkable since there is no free parameter in the theoretical calculation.

9.9.4 GPS Satellite Advance Shift

Many GPS satellites which are orbiting around the earth should be influenced rather heavily by the new gravitational potential. The GPS satellite advance shift can be estimated just in the same way as above, and we obtain

$$\eta = \frac{G^2 M^2}{c^2 R^4 \omega^2} \simeq 1.69 \times 10^{-10} \tag{9.62}$$

where we employ the following values for the GPS satellite [8, 100]

$$R = 2.6561 \times 10^{7}\ \text{m}, \quad M = 5.974 \times 10^{24}\ \text{kg}, \quad \omega = 1.4544 \times 10^{-4} \tag{9.63}$$

since the satellite circulates twice per day. Therefore, the advance shift of the GPS satellite becomes

$$\left(\frac{\Delta T}{T}\right)_{th} \simeq 3.4 \times 10^{-10}. \tag{9.64}$$

This should be compared to the observed value of

$$\left(\frac{\Delta T}{T}\right)_{exp} \simeq 4.5 \times 10^{-10}. \tag{9.65}$$

As seen from the comparison between the calculation and the observed value, the new gravity theory can indeed achieve a remarkable agreement with experiment.

9.9.5 Time Shift of Earth Rotation − Leap Second

Here, we calculate the time shift of the earth rotation around the sun [47]. First, we evaluate the η

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2} \simeq 0.992 \times 10^{-8} \tag{9.66}$$

where we employ the following values for R, M and ω

$$R = 1.496 \times 10^{11} \text{ m}, \ M = 1.989 \times 10^{30} \text{ kg}, \ \omega = 1.991 \times 10^{-7}. \tag{9.67}$$

In this case, we find the time shift for one year

$$(\Delta T)_{th} \simeq 0.621 \ \ s/year \tag{9.68}$$

where $\varepsilon = 0.0167$ is taken. In fact, people have been making corrections for the leap second, and according to the data, they made the first leap second correction in June of 1972. After that, they have made the leap second corrections from December 1972 to December 2008. The total corrections amount to 23 seconds for 36.5 years since we should start from June 1972. This corresponds to the time shift per year

$$(\Delta T)_{exp} \simeq 0.63 \pm 0.02 \ \ s/year \tag{9.69}$$

where the errors are supposed to come from one year shift of the observation. This agrees surprisingly well with the theoretical time shift of the earth.

9.9.6 Observables from General Relativity

Now, we discuss the calculated results by the general relativity [28, 92]. For the Mercury perihelion shift, the result is quite well known, and it can be written in terms of the angular shift. In fact, the angular variable φ is modified by the general relativity to

$$\cos\varphi \longrightarrow \cos(1-\gamma)\varphi \tag{9.70}$$

where γ is found to be

$$\gamma = \frac{3G^2M^2}{c^2R^4\omega^2}. \tag{9.71}$$

This change of the shift in the angular variable could explain the observed Mercury perihelion shift. However, as can be seen from eq.(9.53), this effect vanishes to zero in the case of $\varepsilon = 0$, that is, for the circular orbit. This is, of course, unphysical in that the effect of the general relativity is valid only for the elliptic orbit case. In Newton dynamics, the angular momentum ℓ is the only quantity which can be affected from the external effects like the general relativity or the additional potential.

9.9.7 Prediction from General Relativity

Now, we should calculate the physical observables as to how the general relativity can induce the perihelion shift. In this case, one finds that the change appears in eq.(9.53) as

$$r = \frac{A}{1+\varepsilon\cos\left(\frac{L}{\ell}\varphi\right)} \quad \Rightarrow \quad r = \frac{A}{1+\varepsilon\cos\left((1-\gamma)\varphi\right)} \tag{9.72}$$

and thus the physical observable becomes

$$\omega T \simeq 2\pi(1+2\varepsilon\gamma). \tag{9.73}$$

Therefore, the advance shift of the Mercury perihelion becomes

$$\delta\theta_{th} \equiv \left(\frac{\Delta T}{T}\right)_{th} \simeq 3.3\times 10^{-8} \tag{9.74}$$

which is a factor of 2.5 smaller than the observed value of the Mercury perihelion shift. It should be noted that the predicted shift in eq.(9.72) is indeed the advance shift of the Mercury perihelion as given in eq.(9.73).

In addition, the GPS satellite shift predicted by the general relativity becomes

$$\left(\frac{\Delta T}{T}\right)_{th} \simeq 0.10\times 10^{-10} \tag{9.75}$$

which is very small. This is because the GPS satellite motion has almost the circular orbit around the earth.

Further, the time shift of the earth rotation around the sun predicted by the general relativity becomes

$$(\Delta T)_{th} \simeq 0.031 \;\; s/year. \tag{9.76}$$

This shows that it is much too small compared to the observed time shift of the earth rotation around the sun.

In reality, if the angular momentum is affected from the external potential as given in eq.(9.50), then not only the angular variable but also A in eq.(9.54) should be changed, and therefore as the total effects of the physical observables in the general relativity, eq.(9.73) is modified to

$$\omega T \simeq 2\pi\{1-2(2-\varepsilon)\gamma\} \tag{9.77}$$

which is, unfortunately, a retreat shift since ε is smaller than unity.

9.9.8 Summary of Comparisons between Calculations and Data

We summarize the calculated results of the Mercury perihelion shift, GPS satellite advance shift and Leap Second corrections due to the new gravity model as well as the general relativity. Here, the observed data are compared with the predictions of the model calculations in Table 9.1.

Table 9.1

	Mercury $(\Delta T/T)$	GPS $(\Delta T/T)$	Leap Second ΔT
Observed data	8.0×10^{-8}	4.5×10^{-10}	0.63 ± 0.02 $s/year$
New Gravity	4.8×10^{-8}	3.4×10^{-10}	0.62 $s/year$
General Relativity	3.3×10^{-8}	0.10×10^{-10}	0.031 $s/year$

Table 9.1 shows the calculated results of the Mercury perihelion shift, GPS satellite advance shift and Leap Second corrections together with the observed data. The New Gravity shows the prediction of the new gravity model calculations which are discussed in this paper. The General Relativity is the calculation in which we only consider the angular shift following Einstein. From this table, one sees that the general relativity cannot describe the observed data.

9.9.9 Intuitive Picture of Time Shifts

It may be interesting to note that the velocity of the Mercury or the earth around the sun is one of the fastest objects we can observe as a classical motion. This velocity v is around $v \sim 1.0 \times 10^{-4}\, c$, which leads to the correction of the relativistic effects in physical observables as

$$\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8}$$

which is just the same magnitude as the values observed in the Mercury perihelion shift $(\Delta T/T \sim 5 \times 10^{-8})$ and the leap second corrections $(\Delta T/T \sim 2 \times 10^{-8})$. Therefore, it should not be surprising at all that the new additional gravitational potential which is obtained as the relativistic effects of the gravity potential in Dirac equation can account for the advance shifts of the planets orbiting around the sun.

In this sense, the physical effect of the earth rotation velocity on the perihelion shift can be compared to the Michelson-Morley experiment. The interesting point is that the Michelson-Morley experiment is essentially to examine the kinematical effect of the relativity that the light velocity is not influenced by the earth rotation velocity, even though the classical mechanics indicates it should be affected. On the other hand, the leap second correction is the relativistic effect of the dynamical motion of the earth rotation, and it is a deviation from the Newton mechanics. Both of the observed facts can be understood by the relativistic effects of the earth motion around the sun, and in fact, the Michelson-Morley experiment proves that the light velocity is independent of the speed of the earth rotation, which leads to the concept of the special relativity, while the perihelion shift of the planets confirms the existence of the new additional gravity potential which is derived from the non-relativistic reduction of the Dirac equation with the gravitational potential.

9.9.10 Leap Second Dating

Since we know quite accurately the time shift of the earth rotation around the sun by now, we may apply this time shift to the dating of some archaeological objects such as pyramids or Stonehenge. For example, the time shift of 1000 years amounts to 10.3 minutes, and some of the archaeological objects may well possess a special part of the building which can be pointed to the sun at the equinox. In this case, one may be able to find out the date when this object was constructed. This new dating procedure is basically useful for the stone-made archaeological objects in contrast to the dating of the wooden buildings which can be determined from the Carbon dating. It should be noted that the new dating method has an important assumption that there should be no major earthquake in the region of the archaeological objects.

It should be worthwhile noting that one should be careful for the Leap Second Dating method in the realistic application. This is clear since the earth is also rotating in its own axis when it is rotating around the sun. Therefore, the advance time shift of the earth rotation around the sun should correspond to the retreat time shift of the earth's own rotation if one measures it at one fixed point of the earth surface.