11 磁場と磁性体

11-1 磁場のエネルギー

$$U = \frac{1}{2}\int \mathbf{i}\cdot\mathbf{A}\, d^3r$$

電場の場合

$$U = \frac{1}{2}\int \rho\phi\, d^3r$$

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$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + e\dot{\mathbf{r}}\cdot\mathbf{A} - e\phi$$

$$e\dot{\mathbf{r}} \to \int \mathbf{i}\, d^3r$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{i} \quad より$$

電場

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$U = \frac{1}{2\mu_0}\int \nabla\times\mathbf{B}\cdot\mathbf{A}\, d^3r$$

$$= \frac{1}{2\mu_0}\int\left\{\left[\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right]A_x + \left[\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right]A_y + \left[\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right]A_z\right\} d^3r$$

部分積分をして

surface term $= 0$

(無限遠すなわち)

$$U = \frac{1}{2\mu_0}\int \left[\left(-B_z\frac{\partial A_x}{\partial y} + B_y\frac{\partial A_x}{\partial z}\right) + \left(-B_x\frac{\partial A_y}{\partial z} + B_z\frac{\partial A_y}{\partial x}\right) + \left(-B_y\frac{\partial A_z}{\partial x} + B_x\frac{\partial A_z}{\partial y}\right) \right] d^3r$$

組みかえをすると

$$U = \frac{1}{2\mu_0}\int \left[B_x \overbrace{\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)}^{= B_x} + B_y \overbrace{\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)}^{= B_y} + B_z \underbrace{\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)}_{= B_z} \right] d^3r$$

$$\therefore \quad U = \frac{1}{2\mu_0}\int |\mathbb{B}|^2 \, d^3r$$

磁場のエネルギー