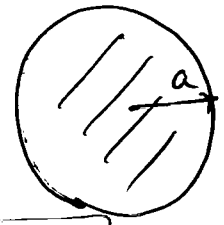
5-5 球状電荷分布の工ネルギー

Q



$\frac{4\pi}{3}\rho a^3 = Q$

電場

$$\begin{cases} E_r = \dfrac{\rho}{3\varepsilon_0} r & (r<a) \\ E_r = \dfrac{Q}{4\pi\varepsilon_0 r^2} & (r>a) \end{cases}$$

$$\begin{cases} \phi(r) = \dfrac{\rho}{6\varepsilon_0}(3a^2 - r^2) & (r<a) \\ \phi(r) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} \end{cases}$$

(i) 電荷密度

$$U = \frac{1}{2}\int \rho\phi \, d^3r$$

$$= \frac{1}{2}\cdot\frac{\rho}{6\varepsilon_0}\int_0^a (3a^2 - r^2)\, 4\pi r^2 dr$$

$$= \frac{4\pi\rho^2}{15\varepsilon_0}a^5 = \frac{3}{20\pi\varepsilon_0}\frac{Q^2}{a} \quad //$$

(ii) 電場による方法

$$U = \frac{\varepsilon_0}{2}\left[\left(\frac{\rho}{3\varepsilon_0}\right)^2\int_0^a r^2 4\pi r^2 dr + \int_a^\infty \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{4\pi r^2}{r^4} dr\right]$$

$$= \frac{3}{20\pi\varepsilon_0}\frac{Q^2}{a} \quad //$$