Relativity : Mistakes in Famous Textbooks

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Preface

There are many articles which make elementary explanations of the theory of relativity since relativity may attract many people indeed. However, it should not be very easy to understand in depth the theory of relativity in a correct fashion. In particular, if authors may explain the theory of relativity following the science history, then there should be a great danger that the interpretation may well be made in the wrong direction.

In this short note, I should like to explain what is the theory of relativity and why it is so important in physics. The essence of the relativity is simple. Two inertial frames which are relatively moving with a constant velocity on the straight line should be equivalent to each other. This means that there is no special inertial system in physics, and therefore, any experiments which are carried out at one inertial frame should produce the same results as those given at the other inertial frame.

Among textbooks which explain the relativity, some of them might insist that the time in the highly moving inertial frame should be delayed compared to the rest frame. However, this is simply wrong, and later I should describe the reason why there is no time delay in any of inertial frames.

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Chapter 1

Relativity

Since the invariance of light velocity is established, people realized that the transformation of inertial frames must be the Lorentz transformation. This Lorentz transformation is derived such that the Maxwell equation should be invariant under the transformation among the inertial frames. Since the transformation is kinematics, there should be no effect on the dynamics from the Lorentz transformation.

1.1 Lorentz Transformation

We should prepare two inertial frames, $R(t, x, y, z)$ and $S(t', x', y', z')$. Now we assume that the frame S is moving with respect to the rest frame $R(t, x, y, z)$ in the $x-$axis with the velocity of v. In this case, the Lorentz transformation can be written as

$$x = \gamma(x' + vt'), \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \qquad y = y', \qquad z = z' \tag{1.1}$$

where γ is defined as

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{1.2}$$

The Lorentz transformation is derived such that the Maxwell equations must have the same shape of differential equations. In eq.(1.1), if the velocity v is much smaller than the light velocity c, then we have

$$x \simeq x' + vt', \qquad t \simeq t', \qquad y = y', \qquad z = z' \tag{1.3}$$

which is just the Galilei transformation.

1.2 Lorentz Transformation of Differential Quantities

Here we should present the Lorentz transformation of differential quantities concerning the coordinates

$$x = \gamma(x' + vt'), \;\; t = \gamma\left(t' + \frac{v}{c^2}x'\right) \tag{1.4}$$

and they can be written as

$$\frac{\partial}{\partial x} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right), \qquad \frac{\partial}{\partial t} = \gamma\left(v\frac{\partial}{\partial x'} - \frac{\partial}{\partial t'}\right). \tag{1.5}$$

Since $y, \; z$ should not be affected, we do not write them here. Now if we define

$$p_x = -i\frac{\partial}{\partial x}, \qquad E = i\frac{\partial}{\partial t} \tag{1.6}$$

then we find

$$p_x = \gamma\left({p_x}' + \frac{vE'}{c^2}\right), \qquad E = \gamma\left(E' + v{p_x}'\right) \tag{1.7}$$

which is just the Lorentz transformation of energy and momentum. Therefore, the scalar product of $px \equiv Et - \boldsymbol{p}\cdot\boldsymbol{r}$ is invariant under the Lorentz transformation

$$px = Et - \boldsymbol{p}\cdot\boldsymbol{r} = p'x' = E't' - \boldsymbol{p}'\cdot\boldsymbol{r}'. \tag{1.8}$$

1.3 Lorentz Invariance of Equation of Motions

The equation of motion should have the same shape of differential equations in any of inertial frames. Therefore, we want to check as to how the Newton equation and Maxwell equation should behave under the Lorentz transformation.

1.3.1 Newton Equation under Lorentz Transformation

As we see from the Lorentz transformation of $x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$, $x,\ t$ **should be independent from each other. But here we assume that** x **should be a function of time. Therefore, the differential of** x **with respect to time** t **becomes**

$$\frac{dx}{dt} = \frac{dx' + vdt'}{dt' + \frac{v}{c^2}dx'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}. \tag{1.9}$$

Further the second derivative should be given as

$$\frac{d^2x}{dt^2} = \frac{1}{\gamma(dt' + \frac{v}{c^2}dx')} d\left(\frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}\right) = \frac{\frac{d^2x'}{dt'^2}}{\gamma^3\left(1 + \frac{v\frac{dx'}{dt'}}{c^2}\right)^3} \neq \frac{d^2x'}{dt'^2} \tag{1.10}$$

which is totally different from the Newton equation. Thus, the Newton equation cannot be invariant under the Lorentz transformation.

1.3.2 Maxwell Equation under Lorentz Transformation

The equation of motion for the electric field of $\boldsymbol{E}$ **can be written when there is no current**

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)\boldsymbol{E} = 0. \tag{1.11}$$

Under the Lorentz transformation, we can prove

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} - \boldsymbol{\nabla}'^2 \tag{1.12}$$

and therefore, the Maxwell equation is invariant under the Lorentz transformation.

Chapter 2

Relativistic Classical Mechanics

In old days, people discussed the relativistic classical mechanics. This is a model which is the extension of classical mechanics to the kinematically relativistic version of mechanics. In terms of science history, this extension should be understandable. However, this is physically a meaningless mechanics which has never been applied to real physics.

2.1 Classical Mechanics

Here, there should be no point to explain any physical meaning of classical mechanics, but it should be important to note that the coordinate x in Newton equation is a function of time t. This is quite strange from the point of view of field theory since t and x should be independent from each other. This is, in fact, also true for the Lorentz transformation.

Then, a question may arise as to why the x should become a function of time in the Newton equation. This can be understood if one looks into the Ehrenfest theorem in detail. One sees that the time dependence of x should be a leftover of time dependence in the state vector $\psi(\boldsymbol{r}, t)$. In this sense, the x of a point particle in classical mechanics is identified as the x in the coordinate system.

2.2 Relativistic Classical Mechanics

The Newton equation can be extended to the relativistic equation of motion by making some modification of kinematics, and this mechanics is called relativistic classical mechanics. Here, however, we do not write this equation of motion since it has no physical meaning at all.

It is clear that we cannot derive the original equation from the equation which is obtained by some approximations. For example, if we take x as a small positive number, we find

$$(1+x)^{\alpha} = 1 + \alpha x + \cdots \tag{2.1}$$

as an approximation. However, it is impossible to guess the l.h.s of equation from the r.h.s. of equation. Thus, it is, of course, clear that the equation of quantum mechanics cannot be obtained from the classical mechanics.

2.3 Definition of Velocity

In classical mechanics, the velocity of particle is defined as

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}. \tag{2.2}$$

However, in the relativistic kinematics, we cannot define the velocity of particle in terms of the derivative of coordinate $\boldsymbol{r}$. This is simply because t and $\boldsymbol{r}$ are independent from each other in the relativistic kinematics as well as in the Lorentz transformation. Therefore, the velocity in the relativistic kinematics can be defined in terms of momentum as

$$\boldsymbol{v} = \frac{\boldsymbol{p}c^2}{E}. \tag{2.3}$$

In the nonrelativistic limit, we obtain, using $E \simeq mc^2$

$$\boldsymbol{v} \simeq \frac{\boldsymbol{p}}{m}. \tag{2.4}$$

This indicates that the velocity of particle in the relativistic kinematics should not be a fundamental physical quantity. In this respect, we cannot define the relativistic classical mechanics in a proper way.

2.3.1 Velocity in Quantum Mechanics

In quantum mechanics, the concept of velocity does not appear in fundamental physical quantities, and the momentum of particle appears as a basic quantity. If one wishes to know the velocity of particle, then one should calculate the expectation value of momentum as

$$\boldsymbol{v} \equiv \frac{1}{m} \int \psi^{\dagger}(\boldsymbol{r}) \hat{\boldsymbol{p}} \psi(\boldsymbol{r}) d^3 r \tag{2.5}$$

where $\hat{\boldsymbol{p}}$ is defined as $\hat{\boldsymbol{p}} = -i\hbar\boldsymbol{\nabla}$ which denotes the momentum operator.

2.3.2 Velocity $\boldsymbol{v}$ in Lorentz Transformation

The velocity of particle is not related to a fundamental physical quantity, but the velocity $\boldsymbol{v}$ of inertial frame appears in the relativity. However, its physical meaning is not very clear yet.

2.4 Lorentz Contraction

People often discuss unphysical effects if they are based on the picture of relativistic classical mechanics. A typical example must be the Lorentz contraction. This is a claim that the length ℓ of the moving frame should be viewed to be contracted if one sees it in the rest frame. But people never explain how it is related to any physical observables.

In reality, however, only the center of mass of length ℓ should be transformed into the other inertial frame, and the Lorentz transformation cannot give any information on the internal structure of length ℓ. In this respect, there is no way to discuss the Lorentz contraction.

Further, the length ℓ should have a finite size, and this means that this ℓ should be a bound state of many atoms. However, in the quantum field theory, there is no way to solve even two body problems in a rigorous fashion. This means that we do not know how to solve the two body problem in the Dirac equation. One may ask a question as to whether the hydrogen atom should be solved in the Dirac equation. However, the hydrogen atom in the Dirac equation is solved with the approximation that proton is sufficiently heavy and thus it should be set to at rest.

2.5 High Energy Heavy Ion Reaction

When a moving inertial frame collides with some targets in the rest frame, is there any possible way to handle the reaction process? In fact, this is just the experiment of high energy heavy ion reactions which were performed around 1980. The energy of heavy ion should amount to around 1 GeV per nucleon, and this should indeed be treated relativistically. The projectile nucleus is ^{4}He, and at Max-Planck Institute, Hüfner and I started to make up some model which can analyze the data of heavy ion reactions.

However, we had some serious problems which are related to the treatment of the projectile wave function of ^{4}He in the rest frame. As one knows, the Lorentz transformation should be only for the center of mass system of ^{4}He, and thus there is no way to obtain the internal wave function of ^{4}He in the rest frame since ^{4}He is moving relativistically. The Lorentz transformation cannot tell us anything about the wave function which has some distribution in space.

As a result, we decided to make analysis of the reaction process in the projectile frame in which the ^{4}He nucleus is at rest. In this way, we are successful in obtaining some important information from the experimental data of this heavy ion reaction.

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Chapter 3

Quantum Field Theory

By now, it is established that the basic theoretical framework is quantum field theory. In particular, the field theory of electron interacting with electromagnetic fields and gravitational field should be the most basic theoretical scheme, and the description of field theory can be found in detail in the textbooks [5, 6].

3.1 Lagrangian Density of QED and Gravity

Here we should write the basic Lagrangian density. This is the system of particle state ψ with the mass m which should interact with electromagnetic field of A_μ as well as the gravitational field $\mathcal{G}$. This Lagrangian density can be written as

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - e\bar{\psi}\gamma^\mu A_\mu \psi - m(1 + g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{v\mu\nu} + \frac{1}{2}\partial_\mu \mathcal{G} \ \partial^\mu \mathcal{G}$$

where $\mathcal{G}$ denotes a massless scalar field. $F^{\mu\nu}$ denotes the field strength and is defined as

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

It should not be very easy to prove that this Lagrangian density is invariant under the Lorentz transformation. But one should

examine it by oneself, and this is the starting point of studying the quantum field theory in depth.

3.2 Calculations in Quantum Field Theory

In quantum field theory, all the calculations are based on the perturbation theory since there is no chance to solve it exactly except free fields. Therefore, it should be important to construct the Dirac vacuum since the solution of Dirac equation contains the negative energy states with the energy eigenvalues which should be physical. However, if there should be negative energy states present, then all the positive energy states should become unstable since the positive energy states should eventually decay into negative energy states.

In order to avoid this difficulty, Dirac proposed and defined the physical vacuum in which all the negative energy states should be occupied. In this case, the vacuum state becomes stable and we have a well-defined quantum field theory for fermions.

3.3 Two Body Problem in Quantum Field Theory

A hydrogen atom consists of proton and electron, and therefore, it is a real two body problem. However, if one wishes to solve the two body Dirac equation, one encounters the difficulty that there is no way to separate the center of mass system from the relative coordinate. Thus, one does not know how to solve the two body Dirac equation in the exact fashion.

Why is it so difficult to solve the Dirac equation for two body problems ? It should not be very easy to find the answer for the above question, but one thing must be clear that the difficulty should be related to the Dirac vacuum state. In terms of quan-

tum field theory, the problem of hydrogen atom may not necessarily be the two body problem, but rather it should be a many body problem. One may say that one sees an electron rotating around proton, but in reality, there should be a small component of electron- positron pair state as a virtual mixture. This is the essence of quantum field theory, and it should naturally and always be a many body problem. Thus, there is no chance to solve it exactly. In physics, we try to understand the behavior of nature with simple equations of motion, and it must be true that physics has made a great success. However, even for the classical mechanics, we cannot solve many body problems. For example, the phenomena of turbulence should be too difficult to understand with physics law. It is extremely interesting to study physics, but we should also understand the limitation of physics application to nature.

Chapter 4

Is Time in Moving Frame Delayed?

From the Lorentz transformation eq.(1.1), it looks that time in the moving frame deviates from the rest frame. However, t and x are variables, and thus, they are not directly related to physical observables. Below we examine whether the time difference of Δt in the Gedanken experiment should be delayed or not.

4.1 Incorrect Gedanken Experiment

Here we first explain the time difference Δt in the Gedanken experiment which is often discussed in the science history. First, we consider a train (moving inertial frame) which is driving in the straight line with a constant velocity v. We assume that there should be big mirror wall in parallel to the straight line with its distance of ℓ.

4.1.1 Time Difference of Moving Frame from Rest Frame

First, an observer in the train emits laser beams against mirror wall. In this case, the observer in the train should not notice that the train is moving. Now this observer should detect the reflected laser beam and should measure the time difference ($2\Delta\tau$). In this

case, we see

$$\ell = c\Delta\tau. \tag{4.1}$$

On the other hand, an observer at the rest frame should detect the laser beam which reflects and travels through the triangle trajectory. In this case, the time difference ($2\Delta t$) should be

$$\sqrt{(c\Delta t)^2 - \ell^2} = v\Delta t. \tag{4.2}$$

Therefore, we find

$$\sqrt{c^2 - v^2}\ \Delta t = c\Delta\tau \tag{4.3}$$

which gives us the following relation between the time differences of $\Delta\tau$ and Δt as

$$\Delta\tau = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta t. \tag{4.4}$$

This suggests that the time difference in the moving frame seems to be somewhat smaller than that of the rest frame.

4.1.2 Time Difference of Rest Frame from Moving Frame

Now we should carry out the same type of Gedanken experiment from the observer at the moving frame. In this case, the rest frame is moving with the velocity of $-v$ for the observer of the moving frame. This can be easily seen if we solve the Lorentz transformation the other way around

$$x' = \gamma(x - vt), \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right), \qquad y' = y, \qquad z' = z. \tag{4.5}$$

Here we see that the rest frame is moving with its velocity of $(-v)$. But otherwise, everything is just the same as in the previous case. In this case, the observer in the rest frame emits laser beams against mirror wall, and the observer in the train should detect the reflected laser beam and should measure the time difference ($2\Delta ct$). Thus, we find

$$\Delta t = \sqrt{1 - \frac{v^2}{c^2}}\, \Delta\tau. \tag{4.6}$$

4.1.3 Inconsistency of Time Difference

What is going on? The results of eqs. (4.4) and (4.6) contradict with each other. Since Δt and $\Delta\tau$ should be observables in the Gedanken experiment, there must be something wrong there.

4.2 Where is Incorrect Process in Gedanken Experiment?

What should be incorrect inductions in the Gedanken experiment? This can be easily seen if we look into eq. (1.1). After t, we took the coordinate of the train as $x' = x + vt$, which is wrong. The correct coordinate after t should be given by the Lorentz transformation as

$$x' = \gamma(x + vt). \tag{4.7}$$

Thus, we should replace in the following way

$$v\Delta t \Longrightarrow \gamma v \Delta t, \qquad c\Delta t \Longrightarrow \gamma c \Delta t. \tag{4.8}$$

Therefore, eq. (4.4) becomes

$$\begin{aligned} \Delta\tau &= \sqrt{1 - \frac{v^2}{c^2}} \times \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\, \Delta t \\ &= \Delta t. \end{aligned}$$

This shows that there is no time delay, and there is no inconsistency. This is just all what we see from the relativity.

4.2.1 No Time Delay in Moving Frame!

From the Gedanken experiment, we see that there is no time delay in the moving frame as compared to the rest frame. This is quite reasonable since the relativity only states that any inertial frames should produce the same results of all physical observables.

Chapter 5

Two Inertial Frames : Examples of Relativity

Here we should discuss possible observables when two inertial frames are involved in physical processes. It should be noted that this consideration is only related to the kinematics, and therefore, we cannot learn anything about dynamics of physical processes.

5.1 Doppler Effect of Light

When a star is moving away from the earth, then lights emitted from this star should be affected by the Lorentz transformation, and this is known as the Doppler effect. Let consider that a star is going away with its velocity v. The momentum p of light emitted at the star should become p' on the earth, and this relation is given by the Lorentz transformation as

$$p' = \gamma\left(p - \frac{vE}{c^2}\right) = \gamma\left(p - \frac{vp}{c}\right) = \frac{p\left(1-\frac{v}{c}\right)}{\sqrt{(1-\frac{v^2}{c^2})}} = p\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}. \tag{5.1}$$

This shows that the momentum of light is decreased. If we express the above relation in terms of wave length, then we obtain

$$\lambda' = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,\lambda. \tag{5.2}$$

Since the wave length of the observed light becomes longer, we call it "red shift". It should be noted that this naming has no physical meaning. It simply says that red light has a longer wave length than that of blue light. The physical reason of the Doppler shift is because the energy and momentum make four dimensional vector, and therefore this is affected by the Lorentz transformation.

5.2 Life Time of Muon Produced in Atmosphere

High energy cosmic ray (protons) may collide with atmospheric N_2 or other molecule and may produce muons with the mass of $m_\mu = 105.6$ MeV/c^2. The life time τ_0 of this lepton is around $\tau_0 \simeq 2\times 10^{-6}$ s. Therefore, muon is unstable. Now a question is as to whether the life time of muon may be affected by the Lorentz transformation or not. This problem is often discussed in science history, but here we should present a right description of muon as to how far it can travel in the air.

Now the life time τ_0 can be written in terms of decay width Γ as

$$\tau_0 = \frac{\hbar}{\Gamma}. \tag{5.3}$$

Here we note that Γ is a Lorentz invariant quantity. Therefore, the life time is also Lorentz invariant, and thus the life time of muon should be the same in any inertial frame.

5.2.1 Travel Distance L of Muon

Now we should calculate the travel distance L of muon after it is created from the collision of protons with atmosphere. This can be evaluated from the Lorentz transformation $x = \gamma(x' + vt')$ as

$$L = \gamma v \tau_0. \tag{5.4}$$

Here we take, as an example, muon with its energy of 1 GeV. In this case, the velocity of muon can be approximated by light velocity of c. The Lorentz factor γ should be $\gamma \simeq 10.6$. Therefore, the value of L becomes

$$L = \gamma v \tau_0 = 10.6 \times 3 \times 10^8 \times 2 \times 10^{-6} \simeq 6.3 \text{ km} \tag{5.5}$$

which is longer by γ than $v\tau_0$. This indicates that the muon produced in the atmosphere may well have some chance to be observed on the earth.

5.2.2 Accelerator Experiment

Unstable particles created by the large accelerator should travel the distance which is given by eq. (5.4). This is longer by a factor of γ than $v\tau_0$, but it has nothing to do with the delay of life time of unstable particles. It is simply due to the Lorentz transformation.

Chapter 6

Conclusions

In the theory of relativity, the transformation property among inertial frames can be given by the Lorentz transformation, and this corresponds to the transformation of a point particle. However, we find some incorrect description of time delay or Lorentz contraction in quite a few text books or short notes. We should stress that the Lorentz transformation is only concerned with the center of mass system of any complex objects, and that is all we know.

In particular, we prove that the time delay of moving frame never occurs in any inertial frames. The basic mistake should be originated from the fact that the coordinate of the moving frame with the velocity of v should be γvt after t second. In the science history, people want to modify v or t in order to understand γvt. However, from the point of theoretical scheme, we should understand γvt by itself in terms of Lorentz transformation.

6.1 Homework Problem

When a train (moving inertial frame) is running with its velocity of v, then the position x of the train after Δt second can be given by the Lorentz transformation as

$$x = \gamma(x' + v\Delta t), \qquad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{6.1}$$

Here we should consider a problem which is concerned with the emission of light in the train. In this case, a question is as to where the light should be found after Δt second in the rest frame.

In this short note, we give the reaching distance ℓ in the rest frame as

$$\ell = \gamma c \Delta t. \tag{6.2}$$

Prove the result of eq.(6.2).

[Hints] :
The light velocity c is not changed when it is emitted from the running train. But in order to understand the range of the light after the emission, we need to consider the Lorentz transformation in a proper manner.

Now, we should take a long train as the moving frame and emit light and observe it after Δt second in the train. In this case, the reaching distance in the moving frame should be simply

$$x' = c\Delta t. \tag{6.3}$$

Therefore, if we make the Lorentz transformation of eq.(6.1), then we find the reaching distance ℓ [(6.2)] of light in the rest frame.

Appendix A

General Relativity

Here, we should make a brief comment on the general relativity. The general relativity is intended to determine the metric tensor $g^{\mu\nu}$ from the second order differential equation, and this means that it is an equation for the coordinate system. However, physics is to understand the motion of particle in the coordinate system which is chosen and set up by the observer. Therefore, the equation for the metric tensor does not mean anything in physics, which is beyond our understanding. In this respect, the Einstein equation is not an equation of motion in physics, even though it is mathematically well defined.

A.1 General Relativity Has No Relation with Gravity

Nevertheless, the general relativity seems to be accepted by quite a few physicists for a long time. What should be reasons for that? We believe that there should be one particular reason why people accepted the general relativity. That must be due to the claim of Einstein that the general relativity should be related to the gravitational theory. Indeed, if we postulate the following equation between the metric tensor $g^{(0\,0)}$ and the gravitational field ϕ

$$g^{(0\,0)} \simeq 1 + 2\phi \tag{A.1}$$

then we find that the general relativity can be related to the gravity.

However, this assumption of eq.(A.1) cannot be justified at all. First, the metric tensor $g^{\mu\nu}$ are unknown functions which should be determined by solving the Einstein equation. Therefore, there is no way to find the shape of metric tensor $g^{(0\,0)}$ in advance. Further, the metric tensor $g^{\mu\nu}$ should be functions of coordinate system, and therefore, there should be no way to relate the $g^{\mu\nu}$ to any dynamical variables like gravitational field ϕ. In this sense, eq.(A.1) is obviously a meaningless equation in physics.

A.2 No Relation of General Relativity to Gravity

It is even simpler to prove that the metric tensor $g^{\mu\nu}$ should have nothing to do with the gravitational field. This can be easily seen if we carefully examine the Einstein equation. The Einstein equation

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G_0 T^{\mu\nu} \tag{A.2}$$

shows that the l.h.s of the equation can be written in terms of Ricci tensor ($R^{\mu\nu}$) which can be described in terms of metric tensor $g^{\mu\nu}$. Therefore, this metric tensor $g^{\mu\nu}$ should be unknown functions which should be determined by solving the Einstein equation.

A.2.1 Who Decided Metric of Right Hand Side?

Firstly, a problem arises as to how the metric of right hand side in the Einstein equation (A.2) should be determined. Probably, the metric must have been assumed to be Minkowski metric. Therefore, the Einstein equation means that the metric tensor $g^{\mu\nu}$ can be determined if the distribution function of stars should be given.

A.2.2 How Can $T^{\mu\nu}$ Be Calculated?

Now the serious problem must be related to a question as to how physical quantities of $T^{\mu\nu}$ can be calculated. The energy-momentum tensor of $T^{\mu\nu}$ can be constructed from the star distributions which should be determined only when the equation of motion of stars under the gravitational field should be properly solved. Therefore, the gravitational field must be assumed there in advance, and thus the metric tensor $g^{\mu\nu}$ has nothing to do with the gravity. This is a clear proof that the general relativity has nothing to do with the gravity as expected.

A.3 No Application of General Relativity to Physics!

Since the general relativity has no relation with the gravity, there is, by now, no way to find and understand any physical meaning of this theory. However, the general relativity has never been used or applied to any area of physics, and thus it has not given rise to any serious problems in physics until now.

A.3.1 Problem of Gravitational Wave

However, sometimes we find a fraction of physicists who make some meaningless claims such as “gravitational wave” or “Black Hole”. This is indeed a serious problem since these physicists did find and get a large amount of science budget. Unfortunately, however, we do not know what we can do for that.

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