# ★ Black Hole for Dilettante ★

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## ★ Black Hole for Dilettante ★

**Even now, some articles about Black Hole are often found in mass media. Unfortunately, however, most of those authors of the articles do not understand the basic physics of Black Hole at all. This must be mainly because they are not theoretical physicists, and further those people that provide news sources of the articles must be, indeed, far from experts not only on quantum field theory but also on general relativity or even physics itself.**

**In this short note, I should make a brief but scientifically correct explanation concerning the physics of Black Hole. At present, there are so many fake information on the physics of Black Hole that some reliable physics lectures must be absolutely needed to improve the present situation. For example, some group of scientists insisted that they discovered Black Hole by making photograph of nucleus of galaxy. This is practically a joke in physics and is quite similar to the story that a man insisted to have seen a god in the forest.**

**Why can these incredible stories be floating around in mass media? This must be because these writers of scientists do not understand the modern physics at all. Nevertheless, if these stories are repeatedly reported in the mass media, then non-physicists might well tend to think and accept that the existence of Black Hole might have been confirmed. This is unfortunate since they do not know what Black Hole should be in terms of correct physics terminology.**

# 1 What is Black Hole ?

Recently, quite a few people have asked me to explain what should be the physics of Black Hole. Therefore, I decided to make a brief but reliable explanation as to what should be the physical meaning of Black Hole in terms of modern physics terminology.

At present, Black Hole is considered to be a kind of star, but its original definition comes from the singularity of the special solution of the Einstein equation. In this sense it has nothing to do with stars. However, those people who consider themselves to be experts on Black Hole may have a picture similar to neutron stars even with higher density. But they believe that light could not escape from the boundary of Black Hole.

• Black Hole in Space-Time :

Now, it is believed that Black Hole is a hole in space-time which is "black", even though they do not understand what it means by "black" in terms of physics. In addition, "experts" on Black Hole do not understand the dynamics of Black Hole at all since they are just physicists who cannot carry out any physical evaluation of neutron stars. In fact, they just talk about Black Hole with their imagination, which has nothing to do with science. Therefore, concerning the story of Black Hole, most of people dilettante are just bound in the chaotic states for a long time.

## 1.1 Nucleus of M87 Galaxy

The recent observation of a would-be Black Hole is related to the nucleus of M87 galaxy. This galaxy is in the distance of 60 million light years from here and it has presumably a diameter of 120 thousand light years. Further it may have a nucleus of galaxy which has a 650 million solar mass. Apart from the accuracy of these numbers, it should be quite natural that the nucleus of galaxy should have some kind of neutron stars, and this is not inconsistent with modern physics. In this case, one can easily estimate the radius of this neutron star, and it is around 10 thousand km which is slightly larger than the earth radius.

## 1.2 Black Hole and Neutron Star

From the kinematics of Black Hole, light cannot escape from the surface of this Black Hole, and this is the basic assumption of Black Hole, though without any physical foundation. This is the only point that is connected to the difference between neutron star and Black Hole. Thus, one can easily see that there is no way to observe this difference between neutron star and Black Hole.

## 1.3 Formation of Super Neutron Star

The formation of super neutron stars should be connected to the large supernova explosion. This type of formation mechanism must be very important to understand, but it has never been studied until now. This may well be connected to the fact that the new gravity model is discovered only about ten years ago [4, 5], and therefore, it is clear that the dynamics of nucleus of galaxy should now be investigated. Indeed, it must be a very interesting subject in nuclear astrophysics.

## 1.4 Responsibility of Person in Charge in Science Section ?

It is a serious problem that mass media published many exaggerated and fake articles which reported that Black Hole was discovered. However, the responsibility for writing incorrect stories of Black Hole may not necessarily be held by people in charge in science section of mass media. But it may well be that the real responsibility of writing wrong articles should rather be taken by the physicists that distributed publicly these incorrect information on Black Hole. Unfortunately, these physicists understand neither modern physics nor general relativity, and probably the fact that they are "physics amusia" must be much more serious than the propagation of wrong information about Black Hole.

## 1.5 Black Hole and Neutron Star

**In order to clarify the physics of Black Hole one must understand quantum field theory, astrophysics, nuclear physics and general relativity in depth, and further one should be able to calculate some physical quantities in this field of research.**

• Gravitational Collapse **?** :

**For example, nuclear physicists should know quite well that the nucleon-nucleon interactions should be strongly repulsive at the short distance, and thus they know that any stars with much higher density than neutron stars cannot exist at all. On the other hand, any large stars with similar density as neutron stars may well exist in nature since there is no basic problem for the formation of gigantic neutron stars. In addition, there is no physical process of gravitational collapse since the gravitational force cannot be very large at the origin of neutron star center. In fact, the gravitational force with finite distribution of mass has the strength which is only 1.5 times stronger at the origin than at the surface.**

# 2 Physics of Black Hole

**Historically, Black Hole is defined as the singularity of the solution in the Einstein equation, and thus it has nothing to do with the formation of stars. Therefore, experts claim that Black Hole is a black "hole" in space-time or they assume that, near the surface of Black Hole, space is distorted so that light cannot escape from Black Hole or something of this kind. These explanations have nothing to do with physics, and therefore only the terminology of Black Hole have been floating around until now. Unfortunately, Black Hole became very popular as if it were a special kind of star.**

## 2.1 Neutron Star

**Experts may explain that Black Hole is a star which has a very high density, and they imagine that it should be similar to neutron stars, but should have even higher density than neutron stars. However, they do not discuss how Black Hole can be formed in the universe since there is no physics equation related to the general relativity. This is clear since the general relativity is a theory for the coordinate system. Therefore, it has nothing to do with dynamics, and indeed no dynamical model is related to the general relativity.**

## 2.2 Nucleus of Galaxy

**Black Hole has no relation with the internal structure of star, and experts define or only claim that Black Hole should have a very large density. However, stars with very high density are, of course, known as neutron stars which are confirmed in terms of Pulsars. In this respect, the nucleus of galaxy should be a very high density star similar to neutron star, and this is consistent with the modern physics. Therefore, it should not be surprising if the nucleus of galaxy becomes an enormous neutron star in size and mass. Indeed, it should attract billions of stars in galaxy, and therefore, it should not be strange at all if there should be a gigantic neutron star at the center of galaxy.**

## 2.3 Surface of Black Hole

The most important assumption concerning Black Hole is related to space distortion at the surface of Black Hole, and it is assumed that light cannot escape from Black Hole. However, space distortion in three dimensions cannot be physically understandable at all. Space distortion is replaced by the light propagation in space, but this cannot be treated in terms of classical mechanics. Further, the general relativity is not a dynamical theory, and therefore, it cannot make any predictions how light should propagate in space. "Experts" on Black Hole only state verbally but not physically as to how space should be distorted, though only from their imagination. The propagation of photon can be treated only if the electromagnetic field is quantized. In addition, space in the general relativity is just the coordinate system, and human being cannot realize real space.

## 2.4 Space Distortion is a Prank of "Physics Amusia"

Thus, nobody can understand space distortion at all, and those people who draw some picture of space distortion are simply making their imagination of scientific fiction. The idea of space distortion must be a result of a prank from "physics amusia", and it has nothing to do with physics.

# 3 Einstein and General Relativity

**Here, there is no important reason to make any tutorial description of the general relativity since the model is worthless in physics. The general relativity is an equation for coordinate system, and Einstein thought that the coordinate system might well be influenced if there should exist a distribution of stars. This is obviously a model which is constructed by physics dilettante. Further, this general relativity is not consistent with the special relativity even though the relativity principle is the most important physics law. Probably, Einstein might have realized this fact of violation of the special relativity, and therefore, he may have claimed that the relativity should be called "special relativity" and his new theory should be named "general relativity".**

## 3.1 Relativity and Its Importance

Most readers may well tend to think that the work of theory of relativity must have been achieved mainly by Einstein. However, it is, by now, known to experts that the credit of constructing the theory of relativity may not necessarily go to Einstein, and his contribution to the relativity should be carefully re-examined.

• Rest Mass :

Indeed, the connection of the rest mass with the Lorentz invariant quantity is an important achievement made by Einstein. However, the real importance of theory of relativity should not be for this rest mass issue, but for the theoretical framework itself that all the theoretical models must satisfy the Lorentz invariance. This is, of course, quite well-known to modern physicists.

In fact, this formulation of relativity is made up by Lorentz and Minkowski and other scientists before Einstein, and therefore, Einstein's contribution to the theory of relativity is not necessarily very great.

• Overvalued :
At present, the work of Einstein concerning theory of relativity is considered to be overvalued. In fact, his paper has no reference, and thus it is written as if everything were done by himself. This is not a fair way of writing papers, but at the time of his day, this way of writing might be one of the reasons of overvaluation.

• General Relativity is Inconsistent with Relativity :
On the other hand, since the general relativity does not satisfy the transformation property of relativity, it is quite difficult to accept that Einstein understands the essence of theory of relativity. In this sense, readers may well understand that it is simply impossible to appreciate the general relativity from modern physics point of view.

## 3.2 Fundamental Equation in Physics

If one wishes to construct a fundamental equation in physics, then one has to make all kinds of careful examinations of physical phenomena from various aspects. However, the Einstein equation is just constructed by making the second order differentials of the metric tensor in the left-hand side and by making the energy- momentum tensor with the distribution function of stars in the right-hand side.

• Physical Ground of Einstein Equation :
Surprisingly and frighteningly, however, there exists no physical phenomenon corresponding to its basic ground of the Einstein equation. Furthermore, one cannot understand what the equation for the coordinate system means in physics. Probably, the author of this equation by himself should not have any concrete pictures for the equation, apart from the vague imagination of space distortion. At the end of 19 century, there seems to be a paper which discusses space distortion, and possibly Einstein may have referred to this paper.

## 3.3 Physics Sense of Einstein

**Up to now, the general relativity is critically reviewed, but no special comment on Einstein himself is made yet. Here, however, I should make a brief comment on Einstein himself even though this is nothing but a feeling. It is not clear whether Einstein might be a "physics amusia" or not, since, at the time of construction of general relativity, quantum mechanics was not discovered yet. Therefore, it is not surprising that he did not have any quantum mechanical and probabilistic way of thinking at the time of 1917.**

• Solvay Conferences and Controversy of Quantum Mechanics :

**However, the controversy of quantum mechanics between Bohr and Einstein at the Solvay Conferences on Physics in 1930 indicates that Einstein could not understand the essence of quantum mechanics which is a probabilistic behavior. This may suggest that Einstein continued to keep the deterministic view of the world, and he wanted to defend the general relativity that is the center of this ideology. By now, it is confirmed that the fundamental physics is described in terms of quantum and probabilistic pictures. On the other hand, some group of physicists still believe in the general relativity, even though it disagrees with quantum behavior. What should be their aim?**

# 4 Physics and Shokunin (Professionals)

**If one wishes to achieve something interesting in physics, then one should become "physics Shokunin". In order to become a theoretical physics Shokunin, for example, one should solve all kinds of exercise problems and examine fundamental physics formulation, in particular, Dirac equation with electromagnetic interactions.**

## 4.1 Importance of Shokunin

**The Shokunin spirit must be important for other area of researches as well. Japanese should have a respect for Shokunin spirit since Edo period. This spirit may well be similar to the Meisterschaft in Germany. In fact, it is believed that this spirit must have been a key issue for the cultural and economic developments of Japan and Germany.**

## 4.2 Drastic Decrease of Theoretical Physics Shokunin

**In the field of theoretical physics, however, Shokunin researchers have decreased drastically. This may be related to the fact that many researchers at present tend to become knowledge-biased, and therefore, they do not work hard to improve their skills in physics. However, even if they transpose a knowledge of one field of research into the other field, this does not mean that they make any real progress in physics. In order to make a solid progress even a little bit, one has to work very hard to improve one's skills of theoretical and computational as much as possible.**

• All Physics Institute :

**At present, a few researchers work at "All Physics Institute" to make a real progress in physics, though they are materially impoverished but physically quite rich indeed. This group of Shokunin researchers are reconstructing modern physics, and a new theoretical scheme will be eventually constructed in near future.**

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# Defect of Einstein's General Relativity

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# Preface

Here I should clarify that there should be no physical meaning in the Einstein equation. This is quite simple, but the proof is exact. There is no ambiguity in this discussion.

In Appendix, I should discuss some old topics which should be reexamined from the point of view of new theoretical scheme. These descriptions of the topics may help young physicists understand modern physics in depth.

# Contents

# Chapter 1

# Einstein's General Relativity

The Einstein equation is a differential equation for the metric tensor of $g^{\mu\nu}$. This metric tensor is defined when the Lorentz invariant quantity $(ds)^2$ is expressed in terms of generalized formula as $(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu$. However, there is no special physical meaning in this generalization, and thus we cannot find any physics related to the metric tensor of $g^{\mu\nu}$. This problem of the general relativity has nothing to do with physics, but it is important in the science history. Therefore, we should explain why the general relativity was accepted to physicists for such a long time, even though it is a meaningless theory in physics.

## 1.1 Relativity Principle

Relativity principle should require that equations of motion in any inertial system should have the same form of differential equations, and, thus, all of the physical observables must be the same in every inertial system. This is the essence of the relativity, and nature can be understood in terms of four basic Lagrangian densities of electromagnetic, weak, strong and gravitational interactions. Indeed, all the field theory models satisfy the relativistic invariance of Lorentz transformation.

### 1.1.1 Lorentz Transformation

Let us consider the moving frame $S(t', x', y', z')$ which is moving with linear motion of constant velocity $v$ along $x-$axis with respect to the rest frame $R(t, x, y, z)$. In this case, the requirement that the equation of motion must be equivalent to each other in both systems can be written in terms of

**Lorentz transformation**

$$x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \quad y = y', \quad z = z'. \tag{1.1}$$

### 1.1.2 Lorentz Invariance

**This Lorentz transformation is the necessary and sufficient condition for relativity principle. However, if we consider only the invariance of Lorentz transformation, then there should be many other physical quantities. Here, we should discuss the small distance square of $(ds)^2$ in four dimensions, which is defined as**

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2.$$

### 1.1.3 Minkowski Space

**This $(ds)^2$ is introduced by Minkowski as a Lorentz invariant quantity**

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \tag{1.2}$$

**which is indeed invariant under the Lorentz transformation of**

$$x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \quad y = y', \quad z = z'. \tag{1.3}$$

**Minkowski extended mathematically $(ds)^2$ to**

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \equiv g^{\mu\nu}dx_\mu dx_\nu \tag{1.4}$$

**even though there is no physical reason for this generalization. In this case, $dx^\mu$ and $dx_\mu$ are introduced as**

$$dx^\mu = (cdt, dx, dy, dz), \quad dx_\mu = (cdt, -dx, -dy, -dz). \tag{1.5}$$

**Further, the metric tensor $g^{\mu\nu}$ is defined as**

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

**This extension of $(ds)^2$ is not incorrect. However, the naming of $g^{\mu\nu}$ as metric tensor is wrong since it is a dimensionless quantity and, therefore, it cannot be taken as any measure of space and time.**

## 1.2 Risk of Generalization

It indeed makes sense that $(ds)^2$ can be taken as a test of Lorentz invariance, and it is also understandable that $(ds)^2$ is expressed in terms of eq.(1.4). However, it should be important to realize that this generalization is physically meaningless since $(ds)^2$ itself is far from any essential quantity in physics.

### 1.2.1 Invariance of $(ds)^2$

Here, we should explain some important point of $(ds)^2$. This $(ds)^2$ is certainly Lorentz invariant, but it is the result of the Lorentz transformation, and not the condition. In fact, there should be many other transformations that can make $(ds)^2$ invariant. This point is quite important since it is related to the essence of relativity. The theory of relativity is a theoretical frame work in which any equation of motion must be the same in any inertial system. The Lorentz transformation satisfies this necessary and sufficient conditions. On the other hand, $(ds)^2$ can serve as a sufficient condition of the relativity requirement, but it is not necessary.

### 1.2.2 Generalized Expression of $(ds)^2$

For a long time, people believed that the generalized expression of $(ds)^2$

$$(ds)^2 = g^{\mu\nu} dx_\mu dx_\nu \tag{1.6}$$

must be basic and essential for $(ds)^2$. This is, of course, an illusion. However, most of physicists may well have been trapped for a long time in a blind state, and this is quite unfortunate.

### 1.2.3 Physical Meaning of $g^{\mu\nu}$

In physics, the expression of (1.2) is essential, and it is impossible to find any physical meaning for the metric tensor of $g^{\mu\nu}$. Indeed, $g^{\mu\nu}$ must be mathematically all right, but it has no physical meaning, and it is just useless.

## 1.3 General Relativity

**Einstein equation is the differential equation for this useless metric tensor $g^{\mu\nu}$, and therefore, we cannot find any physical meaning in this equation.**

**In fact, even if the metric tensor $g^{\mu\nu}$ becomes some function of space and time, there is no effect on the relativity. In case the $(ds)^2$ which is expressed by $g^{\mu\nu}$ in eq.(1.6) has lost the Lorentz invariance, we should make use of $(ds)^2$ as expressed in eq.(1.2). Therefore, there is no physical effect of $g^{\mu\nu}$ in nature at all.**

**This clearly shows that the Einstein equation has nothing to do with physics, and it is simply a mathematical equation which may help young people learn geometrical differential equation as an exercise problem.**

## 1.4 Negative Legacy

**It is a shame that we could not clarify 30 years ago, for example, that the Einstein equation has nothing to do with physics. Many young people wasted their time by learning this general relativity which is completely meaningless in physics. This is quite unfortunate and serious.**

**Incidentally, there was a claim at one point that the Mercury perihelion shifts could be described by the metric tensor which is, by hand, connected to gravity. However, this shift is identified by the discontinuity of Mercury orbit, and, therefore, this prediction is both physically and mathematically meaningless. In this sense, this claim may well be one of the worst theoretical predictions in physics.**

# Appendix A

# Wave Propagation in Medium and Vacuum

The classical wave such as sound can propagate through medium. However, it cannot propagate in vacuum as is well known. This is, of course, clear since the classical wave is the chain of the oscillations of the medium due to the pressure on the density.

On the other hand, quantum wave including photon can propagate in vacuum since it is a particle. Here, we clarify the difference in propagation between classical and quantum waves. The most important point is that the classical wave should be always written in terms of real functions while photon or quantum wave should be described by the complex wave function of the following form of $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ since it should be an eigenstate of the momentum operator.

## A.1 What is Wave ?

The sound can propagate through medium such as air or water. The wave can be described by the following differential equation in one dimension

$$\frac{\partial^2 \phi(x,t)}{\partial t^2} = v^2 \frac{\partial^2 \phi(x,t)}{\partial x^2}. \tag{A.1}$$

where $v$ denotes the speed of wave. The solution of eq.(A.1) is written as

$$\phi(x,t) = A_0 \sin(\omega t - kx) \tag{A.2}$$

where $\omega$ and $k$ denote the frequency and the wave number of the wave, respectively. The dispersion relation of this wave can be written as

$$\omega = vk. \tag{A.3}$$

**Here, it is important to note that the amplitude is written as the real function, in contrast to the free wave function of electron in quantum mechanics. In fact, the free wave function of electron can be written in one dimension as**

$$\psi(x,t) = \frac{1}{\sqrt{V}} e^{i(\omega t - kx)} \tag{A.4}$$

**which is a complex function. The electron can propagate by itself and there is no medium necessary for the electron motion.**

**What is the difference between the real wave amplitude and the complex wave function? Here, we clarify this point in a simple way.**

## A.1.1 Real Wave Function: Classical Wave

**If the amplitude is real such as eq.(A.2), then it can only propagate in medium. This can be clearly seen since the energy of the wave can be transported in terms of the density oscillation which is a real as the physical quantity. In addition, the amplitude becomes zero at some point, and this is only possible when it corresponds to the oscillation of the medium. This means that the wave function of eq.(A.2) has nothing to do with the probability of wave object. Instead, if it is the oscillation of the medium, then it is easy to understand why one finds the zero point of the amplitude. The real amplitude is called a classical wave since it is indeed seen in the world of the classical physics.**

## A.1.2 Complex Wave Function: Quantum Wave

**On the other hand, the free wave function of electron is a complex function, and there is no point where it can vanish to zero. Since this is just the wave function of electron, its probability of finding the wave must be always a finite constant which is, in this case, $\frac{1}{V}$ at any space point of volume $V$.**

## A.2 Classical Wave

The sound propagates in the air, and its propagation should be transported in terms of density wave. The amplitude of this wave can be written in terms of the real function as given in eq.(A.2). This is quite reasonable since the density wave should be described by the real physical quantity. Instead, this requires the existence of the medium (air), and the wave can propagate as long as the air exists. Here, the basic wave equation in one dimension is given in eq.(A.1), and it is similar to the wave equation in quantum mechanics, though it is a real differential equation.

### A.2.1 Classical Waves Carry Their Energy?

In this case, a question may arise as to what is a physical quantity which is carried by the classical wave like sound. It seems natural that the wave carries its energy (or wave length). In fact, the transportation of the energy should be carried out by the compression of the density and successive oscillations of the medium. Therefore, this wave of sound is called compression wave.

### A.2.2 Longitudinal and Transverse Waves

Here, we discuss the terminology of the longitudinal and transverse waves, even though we should not stress its physics too much since there is no special physical meaning.

• Longitudinal wave : The sound propagates as the compressional wave, and the oscillations should be always in the direction of the wave motion. In this case, it is called longitudinal wave. This wave can be easily understood since one can make a picture of the density wave.

• Transverse wave : On the other hand, if the motion of the oscillations is in the perpendicular to the direction of the wave motion, then it is called transverse wave. The tidal wave may be the transverse wave, but its description may not be very simple since the density change may not directly be related to the wave itself.

# A.3 Quantum Wave

**Photon and quantum wave should be quite different from the classical wave, and the quantum wave is a particle motion itself. No medium oscillation is involved. For example, a free electron moves with the velocity $v$ in vacuum, and this motion is also called "wave". The reason why we call it wave is simply due to the fact that the equation of motion that describes electrons looks similar to the classical wave equation of motion. Further, the solution of the wave equation can be described as $e^{ikx}$, and thus it is the same as the wave behavior in terms of mathematics. But the physical meaning of quantum wave is completely different from the classical wave, and the quantum wave is just the particle motion which behaves as the probabilistic motion.**

## A.3.1 Quantum Wave (Electron Motion)

**The wave function of a free electron can be described as**

$$\psi(x,t) = \frac{1}{\sqrt{V}} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} \tag{A.5}$$

**which is a solution of the Schrödinger equation of a free electron,**

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\boldsymbol{\nabla}^2\psi \tag{A.6}$$

**where $k = \sqrt{2m\omega}$, and $V$ denotes the corresponding volume which does not appear in any physical observables. Since the Schrödinger equation is quite similar to the wave equation in a classical sense, one calls the solution of the Schrödinger equation as a wave. However, the physics of quantum wave should be understood in terms of quantum mechanics, and the relation to classical wave should not be stressed too much. That is, the quantum wave is completely different from the classical wave such as sound wave, and one should treat the quantum wave as it is. In addition, the behavior and physics of the classical wave are very complicated, and we do not fully understand the behavior of the classical wave since it involves many body problems in physics.**

### A.3.2 Photon

The electromagnetic wave is called photon which behaves like a particle and also like a wave. This photon can propagate in vacuum and thus it should be considered to be a particle. Photon can be described by the vector potential $\boldsymbol{A}$.

• $\boldsymbol{A}$ is real ! : However, the vector potential $\boldsymbol{A}$ which should correspond to photon is obviously a real function, and therefore, it cannot propagate like a particle. This can be easily seen since the free Hamiltonian of photon commutes with the momentum operator $\hat{\boldsymbol{p}} = -i\boldsymbol{\nabla}$, and therefore it is a simultaneous eigenstate of the Hamiltonian. Thus, the $\boldsymbol{A}$ should be an eigenstate of the momentum operator since the free state must be an eigenstate of momentum. However, any real function cannot be an eigenstate of the momentum operator, and thus the vector field in its present form cannot describe the free particle state of photon.

• Free solution of vector field : What should we do ? The only way of solving this puzzle is to quantize a photon field. First, the solution of $\boldsymbol{A}$ can be written as

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k},\lambda} \frac{1}{\sqrt{2\omega_k V}} \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \left( c^{\dagger}_{\boldsymbol{k},\lambda} e^{-ikx} + c_{\boldsymbol{k},\lambda} e^{ikx} \right) \tag{A.7}$$

with $kx \equiv \omega_k t - \boldsymbol{k}\cdot\boldsymbol{r}$. Here, $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ denotes the polarization vector which will be discussed later more in detail. As one sees, the vector field is indeed a real function.

• Quantization of vector field : Now we impose the following quantization conditions on $c^{\dagger}_{\boldsymbol{k},\lambda}$ and $c_{\boldsymbol{k},\lambda}$

$$[c_{\boldsymbol{k},\lambda},\ c^{\dagger}_{\boldsymbol{k}',\lambda'}] = \delta_{\boldsymbol{k},\boldsymbol{k}'}\delta_{\lambda,\lambda'}, \tag{A.8}$$

$$[c_{\boldsymbol{k},\lambda},\ c_{\boldsymbol{k}',\lambda'}] = 0, \qquad [c^{\dagger}_{\boldsymbol{k},\lambda},\ c^{\dagger}_{\boldsymbol{k}',\lambda'}] = 0. \tag{A.9}$$

In this case, $c^{\dagger}_{\boldsymbol{k},\lambda}$ and $c_{\boldsymbol{k},\lambda}$ become operators. Therefore, we should now prepare the Fock space on which they can operate. This can be defined as

$$c_{\boldsymbol{k},\lambda}|0\rangle = 0 \tag{A.10}$$

$$c^{\dagger}_{\boldsymbol{k},\lambda}|0\rangle = |\boldsymbol{k},\lambda\rangle \tag{A.11}$$

where $|0\rangle$ denotes the vacuum state of photon field. Therefore, if one operates the vector field on the vacuum state, then one obtains

$$\langle \boldsymbol{k},\lambda|\boldsymbol{A}(x)|0\rangle = \frac{1}{\sqrt{2\omega_k V}} \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} e^{-ikx}. \tag{A.12}$$

As one sees, this new state is indeed the eigenstate of the momentum operator and should correspond to the observables. Therefore, photon can be described only after the vector field is quantized. Thus, photon is a particle whose dispersion relation becomes

$$\omega_{\boldsymbol{k}} = |\boldsymbol{k}|. \tag{A.13}$$

## A.4 Polarization Vector of Photon

Until recently, there is a serious misunderstanding for the polarization vector $\epsilon^{\mu}_{\boldsymbol{k},\lambda}$. This is related to the fact that the equation of motion for the polarization vector is not solved, and thus there is one condition missing in the determination of the polarization vector.

### A.4.1 Equation of Motion for Polarization Vector

Now the equation of motion for $A^\mu = (A^0, \boldsymbol{A})$ without any source terms can be written from the Lagrange equation as

$$\partial_\mu F^{\mu\nu} = 0 \tag{A.14}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. This can be rewritten as

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = 0. \tag{A.15}$$

Now, the shape of the solution of this equation can be given as

$$A^\mu(x) = \sum_{\boldsymbol{k}} \sum_{\lambda} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}} \epsilon^{\mu}_{\boldsymbol{k},\lambda} \left[ c_{\boldsymbol{k},\lambda} e^{-ikx} + c^\dagger_{\boldsymbol{k},\lambda} e^{ikx} \right] \tag{A.16}$$

and thus we insert it into eq.(A.15) and obtain

$$k^2 \epsilon^\mu - (k_\nu \epsilon^\nu) k^\mu = 0. \tag{A.17}$$

Now the condition that there should exist non-zero solution of $\epsilon^{\mu}_{\boldsymbol{k},\lambda}$ is obviously that the determinant of the matrix in the above equation should vanish to zero, namely

$$\det\{k^2 g^{\mu\nu} - k^\mu k^\nu\} = 0. \tag{A.18}$$

This leads to $k^2 = 0$, which means $k_0 \equiv \omega_{\boldsymbol{k}} = |\boldsymbol{k}|$. This is indeed a proper dispersion relation for photon.

### A.4.2 Condition from Equation of Motion

Now we insert the condition of $k^2 = 0$ into eq.(A.17), and obtain

$$k_\mu \epsilon^\mu = 0 \tag{A.19}$$

which is a new constraint equation obtained from the basic equation of motion. Therefore, this condition (we call it "Lorentz condition") is most fundamental. It should be noted that the Lorentz gauge fixing is just the same as eq.(A.19). This means that the Lorentz gauge fixing is improper and forbidden for the case of no source term. In this sense, the best gauge fixing should be the Coulomb gauge

$$\boldsymbol{k} \cdot \boldsymbol{\epsilon} = 0 \tag{A.20}$$

from which one finds $\epsilon_0 = 0$, and this is indeed consistent with experiment.

• Number of freedom of polarization vector : Now we can understand the number of degrees of freedom of the polarization vector. The Lorentz condition $k_\mu \epsilon^\mu = 0$ should give one constraint on the polarization vector, and the Coulomb gauge fixing $\boldsymbol{k} \cdot \boldsymbol{\epsilon} = 0$ gives another constraint. Therefore, the polarization vector has only two degrees of freedom, which is indeed an experimental fact.

• State vector of photon : The state vector of photon is already discussed. But here we should rewrite it again. This is written as

$$\langle \boldsymbol{k}, \lambda | \boldsymbol{A}(x) | 0 \rangle = \frac{\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}}{\sqrt{2\omega_k V}} e^{-ikx}. \tag{A.21}$$

In this case, the polarization vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ has two components, and satisfies the following conditions

$$\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \cdot \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda'} = \delta_{\lambda,\lambda'}, \qquad \boldsymbol{k} \cdot \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} = 0. \tag{A.22}$$

### A.4.3 Photon Is Transverse Wave ?

People often use the terminology of transverse photon. Is it a correct expression? By now, one can understand that the quantum wave behaves as a particle motion, and thus it has nothing to do with the oscillation of the medium. Therefore, it is meaningless to claim that photon is a transverse wave. The reason of this terminology may well come from the polarization vector $\epsilon_{\boldsymbol{k},\lambda}$ which is orthogonal to the direction of photon momentum. However, as one can see, the polarization vector is an intrinsic property of photon, and it does not depend on space coordinates.

• No rest frame of photon ! : In addition, there is no rest frame of photon, and therefore, one cannot discuss its intrinsic property unless one fixes the frame. Even if one says that the polarization vector is orthogonal to the direction of the photon momentum, one has to be careful in which frame one discusses this property.

In this respect, it should be difficult to claim that photon behaves like a transverse wave. Therefore, one sees that photon should be described as a massless particle which has two degrees of freedom with the behavior of a boson. There is no correspondence between classical waves and photon, and even more, there is no necessity of making analogy of photon with the classical waves.

## A.5 Poynting Vector and Radiation

**Here, we discuss the Poynting vector how it appears in physics, and show that it cannot propagate in vacuum at all, and thus it has nothing to do with radiations. Also, we present a brief description of the basic radiation mechanism how photon can be emitted.**

### A.5.1 Field Energy and Radiation of Photon

**Before discussing the propagation of Poynting vector, we should first discuss the mechanism of the radiation of photon in terms of classical electrodynamics. The interaction Hamiltonian can be written as**

$$H_I = -\int \boldsymbol{j} \cdot \boldsymbol{A}\, d^3r \tag{A.23}$$

**which should be a starting point of all the discussions. Now, we make a time derivative of the interaction Hamiltonian and obtain**

$$W \equiv \frac{dH_I}{dt} = -\int \left[ \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A} + \boldsymbol{j} \cdot \frac{\partial \boldsymbol{A}}{\partial t} \right] d^3r. \tag{A.24}$$

**Since we can safely set $A^0 = 0$ in this treatment, we find**

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}. \tag{A.25}$$

**Therefore, we can rewrite eq.(A.24) as**

$$W = \int \boldsymbol{j} \cdot \boldsymbol{E}\, d^3r - \int \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A}\, d^3r. \tag{A.26}$$

**Defining the first term of eq.(A.24) as $W_E$, we can rewrite $W_E$ as**

$$W_E \equiv \int \boldsymbol{j} \cdot \boldsymbol{E}\, d^3r = -\frac{d}{dt} \left[ \int \left( \frac{1}{2\mu_0} |\boldsymbol{B}|^2 + \frac{\varepsilon_0}{2} |\boldsymbol{E}|^2 \right) d^3r \right] - \int \boldsymbol{\nabla} \cdot \boldsymbol{S}\, d^3r \tag{A.27}$$

**which is just the energy flow of electromagnetic fields.**

### A.5.2 Poynting Vector

Here, the last term of eq.(A.27) is Poynting vector $\boldsymbol{S}$ as defined by

$$\boldsymbol{S} \equiv \boldsymbol{E} \times \boldsymbol{B} \tag{A.28}$$

which is connected to the energy flow of the electromagnetic field. This Poynting vector is a conserved quantity, and thus it has nothing to do with the electromagnetic wave. In addition, it is a real quantity, and thus there is no way that it can propagate in vacuum. In addition, the Poynting vector cannot be a target of the field quantization, and thus it always remains classical since it is written in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$. However, there is still some misunderstanding in the textbooks on Electromagnetism, and thus, one should be careful for the treatment of the Poynting vector.

• Exercise problem: Here, we present a simple exercise problem of circuit with condenser with $C$ (disk radius of $a$ and distance of $d$) and resistance with $R$. The electric potential difference $V$ is set on the circuit. In this case, the equation for the circuit can be written as

$$V = R\frac{dQ}{dt} + \frac{Q}{C}.$$

This can be easily solved with the initial condition of $Q = 0$ at $t = 0$, and the solution becomes

$$Q = CV\left(1 - e^{-\frac{t}{RC}}\right).$$

Therefore, the electric current $J$ becomes

$$J = \frac{dQ}{dt} = \frac{V}{R}e^{-\frac{t}{RC}}.$$

In this case, we find the electric field $\boldsymbol{E}$ and the displacement current $\boldsymbol{j}_d$

$$\boldsymbol{E} = \frac{Q}{\pi a^2}\boldsymbol{e}_z = \frac{VC}{\varepsilon_0 \pi a^2}\left(1 - e^{-\frac{t}{RC}}\right)\boldsymbol{e}_z \tag{A.29}$$

$$\boldsymbol{j}_d = \frac{\partial \boldsymbol{E}}{\partial t} = \frac{V}{R\pi a^2}e^{-\frac{t}{RC}}\boldsymbol{e}_z. \tag{A.30}$$

Thus, the magnetic field $\boldsymbol{B}$ becomes

$$\boldsymbol{B} = \frac{i_d\, r}{2}\boldsymbol{e}_\theta = \frac{r}{2\pi a^2 R}\; e^{-\frac{t}{RC}}\;\boldsymbol{e}_\theta$$

where $\int_C \boldsymbol{B} \cdot d\boldsymbol{r} = \mu_0 i_d \pi r^2$ is used. Therefore, the Poynting vector at the surface (with $r = a$ ) of the cylindrical space of the disk condenser becomes

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B} = -\frac{V^2}{2\pi a R d}\, e^{-\frac{t}{RC}} \left(1 - e^{-\frac{t}{RC}}\right) \boldsymbol{e}_r.$$

It should be noted that the energy in the Poynting vector is always flowing into the cylindrical space. Therefore, the electric field energy is now accumulated in the cylindrical space. There is, of course, no electromagnetic wave radiation, and in fact, the Poynting vector is the flow of field energy, and has nothing to do with the radiation of photon.

### A.5.3 Emission of Photon

The emission of photon should come from the second term of eq.(A.26) which can be defined as $W_R$, and thus

$$W_R = -\int \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A}\, d^3r. \tag{A.31}$$

In this case, we can calculate the $\frac{\partial \boldsymbol{j}}{\partial t}$ term by employing the Zeeman effect Hamiltonian with a uniform magnetic field of $\boldsymbol{B}_0$

$$H_Z = -\frac{e}{2m_e} \boldsymbol{\sigma} \cdot \boldsymbol{B}_0. \tag{A.32}$$

The relevant Schrödinger equation for electron with its mass $m_e$ becomes

$$i\frac{\partial \psi}{\partial t} = -\frac{e}{2m_e} \boldsymbol{\sigma} \cdot \boldsymbol{B}_0\, \psi. \tag{A.33}$$

Therefore, we find

$$\frac{\partial \boldsymbol{j}}{\partial t} = \frac{e}{m_e}\left[\frac{\partial \psi^\dagger}{\partial t}\hat{\boldsymbol{p}}\psi + \psi^\dagger \hat{\boldsymbol{p}}\frac{\partial \psi}{\partial t}\right] = -\frac{e^2}{2m_e^2} \boldsymbol{\nabla} B_0(\boldsymbol{r}). \tag{A.34}$$

In order to obtain the photon emission, one should quantize the field $\boldsymbol{A}$ in eq.(A.31).

• Field quantization : The field quantization in electromagnetic interactions can be done only for the vector potential $\boldsymbol{A}$. The electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are classical quantities which are defined before the field quantization.

## A.6 Gravitational Wave

People often discuss the gravitational wave which is supposed to come from the Einstein equation. In this case, one sees that the equation for the metric tensor is all real, and thus the solution of this equation must be also real. Therefore, the gravitational wave, if at all exists, is a real function, and thus, it cannot propagate in vacuum unless one believes the aether hypothesis.

• No quantization of gravity : In addition, there is no physical meaning to quantize the metric tensor, and therefore, there is no chance that the gravitational wave propagates in vacuum.

### A.6.1 General Relativity

Since we treat the gravitational wave, we should make a brief comment on the general relativity. Einstein invented the Einstein equation which is the second order differential equation for the metric tensor $g^{\mu\nu}$. The Einstein equation is written as

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G_0 T^{\mu\nu} \tag{A.35}$$

where $R^{\mu\nu}$ is called Ricci tensor and is written in terms of second order differential of $g^{\mu\nu}$. $T^{\mu\nu}$ denotes the energy-momentum tensor which can be expressed by the distribution function of stars. Note that the energy-momentum tensor can be only defined when the distribution of stars is introduced. Classical particles cannot make the energy-momentum tensor since it is normally defined for quantum fields.

A question may arise as to why the general relativity can be related to the gravitational theory. This reason is simply because Einstein claimed that the gravitational Poisson equation should be derived from the general relativity at the weak gravitational limit. However, in his proof, he assumed the following strange equation

$$g^{00} \simeq 1 + 2\phi \tag{A.36}$$

where $\phi$ denotes the gravitational field. Because of this equation (A.36), he could derive the gravitational Poisson equation

$$\boldsymbol{\nabla}^2 \phi(\boldsymbol{r}) = 4\pi G \rho(\boldsymbol{r}) \tag{A.37}$$

where $G$ and $\rho$ denote the gravitational constant and the density, respectively.

• Eq.(A.36) is correct ? :
Here, we show that eq.(A.36) is not only strange but simply incorrect. In order to do so, we should examine the physical meaning of the equation $g^{00} \simeq 1 + 2\phi$. We should notice that 1 (unity) in the right hand side of eq.(A.36) is a simple number. This is clear since the metric tensor is just the coordinate system itself. However, the gravitational field $\phi$ is a dynamical variable, and therefore this summation of two different categories is simply meaningless.

• No connection between general relativity and gravity :
By now it should be clear that the general relativity has nothing to do with gravity. It is a theory for the coordinate system (metric tensor), but it is not a theory to describe nature.

A rigorous proof that the metric tensor of $g^{\mu\nu}$ has nothing to do with gravity can be made in the following way. If we look at the Einstein equation [eq.(A.35)], this is a differential equation to determine the metric tensor that appears in the left hand side of eq.(A.35). On the other hand, the right hand side of eq.(A.35) consists of the energy momentum tensor which should be made from star distribution functions. However, the star distribution can be determined only after the distribution of stars should be solved with gravitational potential. Therefore, before determining the metric tensor, we must assume the gravitational force in advance, and thus the metric tensor should never become a function of gravity. Therefore, eq.(A.36) is simply incorrect.

### A.6.2 Gravitational Wave

Since the general relativity has nothing to do with gravity, there is no chance to connect the gravitational wave to the general relativity. Further, as we see later, the gravity is not quantized, and therefore, there is no concept of gravitational wave in physics at all.

# Appendix B

# New Gravity Model

**Quantum field theory is based on the free Dirac fields and four fundamental interactions. These are electromagnetic, weak, strong and gravitational interactions. In terms of coupling constant, the electromagnetic interaction must be a standard, and the strength of the coupling constant which is dimensionless is found to be**

$$\alpha = \frac{1}{137}. \tag{B.1}$$

**On the other hand, the strong interaction should be stronger by two orders of magnitude than the electromagnetic interaction while weak interaction must be weaker by a few orders of magnitude than the electromagnetic interaction. In this respect, the gravity is, by far, the weakest force among the four interactions. In fact, the gravity is by the order of $\sim 10^{-30}$ smaller than the electromagnetic interaction.**

## B.1 Introduction

**Nevertheless, the gravity is very important in the universe for the formation of stars and galaxies since the force has a very long range, and it is always attractive. In fact, apart from strong interactions that should responsible for nuclear fusion in stars, the basic ingredients of forming stars and galaxies in the universe should be the gravitational interaction.**

### B.1.1 Why Gravity Has Large Effects on Star Formation?

The gravity is crucially important for the formation of stars even though the interaction strength is quite weak. There are two important aspects in the gravity when the stars should be formed. The first point is connected to the interaction range which is very long since it has the shape of $1/r$. The other point is that the gravity is always attractive and the strength of the force should be proportional to the masses of interacting objects. Therefore, as long as the corresponding body is massive, there should exist the attractive interactions from all other massive objects even though they are far away from each other. Because of the attractive nature, there should be no shielding in contrast to the electromagnetic cases.

### B.1.2 Dirac Equation with Gravitational Potential

When the energy of a particle becomes as high as its mass, then we have to consider the relativistic equation of motion under the gravitational potential. In this case, the Newton equation is not appropriate for describing a relativistic motion, and thus, we have to find a new equation of motion. Since we know that the classical mechanics is derived from the Schrödinger equation, we should start from the relativistic equation in quantum mechanics. This is the Dirac equation, and therefore, we have to consider the Dirac equation with the gravitational interaction.

However, the Dirac equation with the gravitational potential has not been determined properly for a long time. This problem is connected to the ambiguity as to whether the gravitational potential should be taken as the fourth component of the vector type interaction or the mass term of scalar type interaction. This problem was not settled until recently, and thus, we should consider the gravitational field theory in some way or other. As will be discussed later, the new gravity model is, indeed, constructed in terms of a massless scalar field theory. Therefore, the corresponding Dirac equation with the gravitational potential is well established by now [3, 17].

## B.2 Dirac Equation and Gravity

The Newton equation works very well under the gravitational potential, and indeed, the Kepler problem is best understood by solving the Newton equation.

• Ehrenfest Theorem :
This Newton equation itself is obtained from the Schrödinger equation by making some approximation such as Ehrenfest theorem. In this case, the time development of the expectation values of $\boldsymbol{r}$ and $\boldsymbol{p}$ in quantum mechanics lead to the Newton equation.

• Foldy-Wouthuysen Transformation :
The Schrödinger equation can be derived from the Dirac equation by making the Foldy-Wouthuysen transformation which is a unitary transformation. Therefore, the Dirac equation must be the starting point from which the Newton equation can be derived.

### B.2.1 Dirac Equation and Gravitational Potential

As can be seen from the present discussion, it should be crucially important to have the Dirac equation with the gravitational potential properly taken into account. Otherwise, we cannot obtain the Newton equation with the gravitational potential. In other words, we should not start from the Newton equation with the gravitational potential since it is obtained only after some series of approximations should be properly made for quantum mechanics.

• Dirac Equation with Coulomb Potential :
Before going to the discussion of the Dirac equation with the gravity, we should first discuss the Dirac equation with the Coulomb potential of $V_c(r) = -\frac{Ze^2}{r}$. This is well-known and can be written as

$$\left(-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + m\beta - \frac{Ze^2}{r}\right)\Psi = E\Psi. \tag{B.2}$$

On the other hand, we should be careful in which way we put the gravitational potential of $V(r) = -\frac{GmM}{r}$ into the Dirac equation since there are two different ways, either the same way as the Coulomb case or putting the gravitational potential into the mass term.

**• Dirac Equation with Gravitational Potential :**
**In fact, the right Dirac equation with the gravitational potential of $V(r) = -\frac{GmM}{r}$ can be written by putting it into the scalar term as**

$$\left[ -i\boldsymbol{\nabla} \cdot \boldsymbol{\alpha} + \left( m - \frac{G_0 mM}{r} \right) \beta \right] \Psi = E\Psi. \tag{B.3}$$

**This is obtained from the field theoretical construction of the gravity model. By now, we see that the scalar type potential of gravity must be the right gravitational potential, and we should discuss it more in detail below.**

## B.3 New Gravity Model

**When we wish to construct the theory of gravity, the first thing we should work out should be to find the framework in which the gravitational potential can be properly taken into account in the Dirac equation. Without doing this procedure, there should be no way to consider the theory of gravity. In fact, the Dirac equation for a particle with its mass $m$ in the gravitational potential can be written as**

$$\left[ -i\boldsymbol{\nabla} \cdot \boldsymbol{\alpha} + \left( m - \frac{GmM}{r} \right) \beta \right] \Psi = E\Psi \tag{B.4}$$

**where $M$ denotes the mass of the gravity center. In addition, if we make the non-relativistic reduction using the Foldy-Wouthuysen transformation, then we find the gravitational potential in classical mechanics**

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2} \left( \frac{GmM}{r} \right)^2 \tag{B.5}$$

**where the second term of the right hand side should be the additional potential which appears as the relativistic effect. This additional potential of gravity is a new gravitational potential, and this must be a new discovery ever since nineteenth century. It turns out that this new potential can explain the problem of leap second of the earth revolution period which will be discussed later.**

**• Rough Estimation of Relativistic Effect :**
Historically, the first check of the relativistic effect was done by Michelson-Morley using the velocity of the earth revolution which should be the fastest object relevant to the observed speed on the earth. The result of Michelson-Morley experiment showed that the speed of light is not affected by the earth revolution, and this leads to the concept of the relativity principle. The relativistic effect in this case is

$$\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8} \tag{B.6}$$

where $c$ and $v$ denote the velocities of light and the earth revolution, respectively. It should be interesting to note that the leap second of the earth revolution period is found to be ($\Delta T/T \sim 2 \times 10^{-8}$) which is just the same order of magnitude as the relativistic effect.

## B.3.1 Lagrangian Density

When we consider the theory of gravity, we should start from the scalar field theory since it gives always attractive interactions.

**• Lagrangian Density of Gravity :**
Here, we should write the Lagrangian density of a fermion field $\psi$ interacting with the electromagnetic field $A_\mu$ and the gravitational field $\mathcal{G}$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - m(1+g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\upsilon\mu\nu} + \frac{1}{2}\partial_\mu\mathcal{G}\ \partial^\mu\mathcal{G} \tag{B.7}$$

where $m$ denotes the fermion mass. The gravitational field $\mathcal{G}$ is a massless scalar field. The reason why people did not consider the scalar field for the gravity should be mainly because the scalar field should not be renormalizable. However, there is no necessity of the field quantization of the gravitational field, and thus, there is no divergence at all.

**• Gravity Cannot Be Gauge Theory :**
For a long time, people believed that the basic field theory must be a gauge theory, even though there is no foundation for this belief. Indeed, the gauge theory has both attractive and repulsive interactions, and therefore, it is clear that this model of gauge field theory should not be suitable for the gravity.

By now, it is known that only the gauge theory of quantum electrodynamics using the Feynman propagator should give rise to some divergences in the calculation of physical observables such as vertex corrections. In fact, there is no divergence for the vertex corrections which are calculated from the massive vector field theory [3].

### B.3.2 Equation for Gravitational Field

From the Lagrangian density, we can obtain the equation for the gravitational field from the Lagrange equation. Here, we can safely make the static approximation for the equation of motion, and obtain the equation for the gravitational field $\mathcal{G}_0$ as

$$\boldsymbol{\nabla}^2 \mathcal{G}_0 = mg\rho_g \tag{B.8}$$

where $m\rho_g$ corresponds to the matter density. The coupling constant $g$ is related to the gravitational constant $G$ as

$$G = \frac{g^2}{4\pi}.$$

This equation eq.(B.8) is indeed the Poisson equation for gravity.

### B.3.3 Dirac Equation with Gravitational Potential

From the Lagrangian density with gravity and electromagnetic interactions, we can derive the Dirac equation

$$\left[ -i\boldsymbol{\nabla} \cdot \boldsymbol{\alpha} + m\beta \left(1 + g\mathcal{G}\right) - \frac{Ze^2}{r} \right] \Psi = E\Psi. \tag{B.9}$$

Further, in case the gravitational force is produced by nucleus with its mass of $M$, the Dirac equation becomes

$$\left[ -i\boldsymbol{\nabla} \cdot \boldsymbol{\alpha} + \left( m - \frac{GmM}{r} \right) \beta - \frac{Ze^2}{r} \right] \Psi = E\Psi \tag{B.10}$$

which is just the equation discussed in the previous section.

### B.3.4 Foldy-Wouthuysen Transformation of Dirac Hamiltonian

**The Dirac equation with the gravitational interaction**

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+\left(m-\frac{GmM}{r}\right)\beta\right]\Psi=E\Psi \tag{B.11}$$

**can be reduced to the non-relativistic equation in quantum mechanics. This can be done in terms of Foldy-Wouthuysen transformation which is a unitary transformation. Therefore, the transformation procedure is very reliable indeed.**

**• Foldy-Wouthuysen Transformation :**
**Here, we start from the Hamiltonian with the gravitational potential**

$$H=-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+\left(m-\frac{GmM}{r}\right)\beta. \tag{B.12}$$

**This Hamiltonian can be rewritten in terms of the Foldy-Wouthuysen transformation which is somewhat a complicated and tedious procedure involved, though it can be done in a straightforward way [26]. In this case, the non-relativistic Hamiltonian should be obtained as**

$$H=m+\frac{\boldsymbol{p}^2}{2m}-\frac{GmM}{r}+\frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2-\frac{1}{2m^2}\frac{GMm}{r^3}(\boldsymbol{s}\cdot\boldsymbol{L}) \tag{B.13}$$

**which is kept only up to the order of** $\left(\frac{\boldsymbol{p}}{m}\right)^2\frac{GM}{r}$**.**

### B.3.5 Classical Limit of Hamiltonian with Gravity

Here, we should calculate the classical equation of motion from the non-relativistic Hamiltonian in quantum mechanics. In this case, the Hamiltonian which is only relevant to the present discussion can be written as

$$H = \frac{\boldsymbol{p}^2}{2m} - \frac{GmM}{r} + \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2. \tag{B.14}$$

This can be reduced to the Newton equation by making the expectation values of operators in quantum theory in terms of the Ehrenfest theorem. In this case, we approximate the products by the factorization in the following way

$$\left\langle \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2 \right\rangle = \left\langle \frac{1}{2m^2}\frac{GmM}{r} \right\rangle \left\langle \boldsymbol{p}^2 \right\rangle \tag{B.15}$$

which must be a good approximation in the classical mechanics application. In addition, we make use of the Virial theorem

$$\left\langle \frac{\boldsymbol{p}^2}{m} \right\rangle = -\left\langle V \right\rangle . \tag{B.16}$$

Therefore, we finally obtain the following additional potential

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2 \tag{B.17}$$

which is a new gravitational potential in classical mechanics. The derivation of the additional potential is similar to the Zeeman effects in that both interactions appear in the non-relativistic reduction as the higher order terms of coupling constant.

## B.4 Predictions of New Gravity Model

By now, a new gravity model is constructed, and as a byproduct, there appears the additional gravitational potential. This is a very small term, but its effect can be measurable. Indeed, this is the relativistic effect which becomes

$$\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8} \tag{B.18}$$

for the earth revolution around the sun. On the other hand, the leap second of the earth revolution is found to be

$$\left(\frac{\Delta T}{T}\right) \sim 2 \times 10^{-8} \tag{B.19}$$

which is just the same order of magnitude as the relativistic effect. Therefore, as we see later, it is natural that the leap second value can be understood by the additional potential of the new gravity model.

### B.4.1 Period Shifts in Additional Potential

In the new gravity model, there appears the additional potential in addition to the normal gravitational potential. In the case of the earth revolution around the sun, this potential is written as

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2 \tag{B.20}$$

where the second term is the additional potential [3]. Here, $G$ and $c$ denote the gravitational constant and the velocity of light, respectively. $m$ and $M$ correspond to the masses of the earth and the sun, respectively.

• Non-integrable Potential :
It should be important to note that the additional potential should be a non-integrable, and therefore, the treatment should be done in terms of the perturbation theory. In this case, the Newton equation with the perturbative procedure of the additional potential can be solved, and the period $T$ of the revolution is written as

$$\omega T \simeq 2\pi(1 + 2\eta) \tag{B.21}$$

where $\eta$ is given as

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2}. \tag{B.22}$$

Here, $R$ is the average radius of the earth orbit. The angular velocity $\omega$ is related to the period $T$ by

$$\omega = \frac{2\pi}{T}. \tag{B.23}$$

The period shift due to the additional potential becomes

$$\frac{\Delta T}{T} = 2\eta \tag{B.24}$$

which is the delay of the period of the revolution [3, 17].

### B.4.2 Period Shifts of Earth Revolution (Leap Second)

In the earth revolution, the orbit radius, the mass of the sun and the angular velocity can be written as

$$R = 1.496 \times 10^{11} \text{ m}, \quad M = 1.989 \times 10^{30} \text{ kg}, \quad \omega = 1.991 \times 10^{-7}. \tag{B.25}$$

In this case, the period shift becomes

$$\frac{\Delta T}{T} = 2\eta \simeq 1.981 \times 10^{-8}. \tag{B.26}$$

Therefore, the period of the earth revolution per year amounts to

$$\Delta T_{N.G.} = 0.621 \quad \text{[s/year]} \tag{B.27}$$

which is a delay. This suggests that the corrections must be necessary in terms of the leap second.

• Leap Second :
In fact, the leap second corrections have been made for more than 40 years. The first leap second correction started from June 1972, and for 40 years, people made corrections of 25 second. Therefore, the average leap second per year becomes

$$\Delta T^{Obs}_{N.G.} \simeq 0.625 \pm 0.013 \quad \text{[s/year]} \tag{B.28}$$

which agrees perfectly with the prediction of eq.(B.27).

• Definition of Newcomb Time :
Newcomb defined the time series of second in terms of the earth revolution period. However, the recent measurement of time in terms of atomic clock turns out to deviate from the Newcomb time [24]. This deviation should be due to the relativistic effects, and indeed this deviation can be understood by the additional potential of gravity.

### B.4.3 Mercury Perihelion Shifts

For a long time, people believed that the Mercury perihelion shifts can be understood by the higher order effects of general relativity. However, it is proved that there should be no perihelion shifts for one period of the earth revolution.

Instead, there should be the Mercury perihelion shifts which may arise from the effects of other planets such as Jupiter if we can measure the perihelion shifts for some long period of revolutions. Concerning the Mercury perihelion shifts, however, the measurements as well as the calculations of the effects from other planets should be carried out more carefully. After the calculation of Newcomb in the 19 century, no careful calculation on the perihelion shifts has been done until now.

### B.4.4 Retreat of Moon

The moon is also affected by the additional potential of gravity from the earth. The shifts of the moon orbit can be expressed just in the same way as the earth revolution. In this case, $\eta$ can be written as

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2}. \tag{B.29}$$

Here, $R$ is the radius of the moon orbit. $M$ and $\omega$ denote the mass of the earth and the angular velocity, respectively. They are written as

$$R = 3.844 \times 10^8 \ \ \text{m}, \quad M = 5.974 \times 10^{24} \ \ \text{kg}, \quad \omega = 2.725 \times 10^{-6} \tag{B.30}$$

Therefore, the period shift becomes

$$\frac{\Delta T}{T} = 2.14 \times 10^{-11}. \tag{B.31}$$

Now, we should carry out the calculation as to how the orbit can be shifted, and the shift of the angle can be written as

$$\Delta\theta = 4\pi\eta. \tag{B.32}$$

Thus, the orbit shift $\Delta\ell_m$ can be written as

$$\Delta\ell_m = R\Delta\theta \simeq 0.052 \ \ \text{m} \tag{B.33}$$

and therefore, the shift per year becomes

$$\Delta\ell_{m \ (one \ year)} = \Delta\ell_m \times \frac{3.156 \times 10^7}{2.36 \times 10^6} \simeq 69.5 \ \ \text{cm}. \tag{B.34}$$

**• Calculated Results of Retreat of Moon :**
**Since the orbit of the moon is ellipse, the orbit shift can be seen as if it were retreated [27]. The orbit is described by**

$$r = \frac{R}{1 + \varepsilon \cos\theta}. \tag{B.35}$$

**In addition, the eccentricity is quite small ($\varepsilon = 0.055$) and therefore, we can rewrite the above equation as**

$$r \simeq R(1 - \varepsilon \cos\theta). \tag{B.36}$$

**Thus, the orbit shift $\Delta r$ at $\theta \simeq \frac{\pi}{2}$ becomes per year**

$$\Delta r \simeq R\Delta\theta\ \varepsilon \simeq \Delta\ell_{m\ (one\ year)}\ \varepsilon\ \simeq\ 3.8\ \ \text{cm} \tag{B.37}$$

**On the other hand, the observed value of the retreat shift of the moon orbit is**

$$\Delta r_m^{obs} \simeq 3.8\ \ \text{cm} \tag{B.38}$$

**which agrees very well with the prediction.**

**• Retreat Shift is not Real! :**
**It should be noted that this observation is only possible by making use of the Doppler shift measurement. This is not a direct measurement of the moon orbit distance which is not possible due to the uncertainty of the accuracy of light velocity**

$$c = (2.99792458\ \pm 0.000000012) \times 10^8\ \ \text{cm/s}. \tag{B.39}$$

**The accuracy of the orbit shift $\Delta r_m^{obs} \simeq 3.8$ cm is at the order of $10^{-10}$ while the light velocity is measured only up to $10^{-8}$ accuracy. This means that the shift of the orbit radius is just the instantaneous and apparent effect.**

## B.5 Summary

The new gravity theory of eq.(B.7)) can naturally lead to the Dirac equation of eq.(B.3). This is very important in modern physics since we have now the Dirac equation with the gravitational potential properly taken into account. This Dirac equation can be reduced to the non-relativistic Hamiltonian which then gives rise to the Newton equation with the gravitational potential, and this new equation should contain a new gravitational potential as the additional potential.

• Massless Scalar Field :
The fact that the gravity is described by the massless scalar field can give rise to some important effects on the non-relativistic reduction. This is in contrast to the Coulomb case, but rather similar to the non-relativistic reduction of the vector potential case. In the non-relativistic reduction of the vector potential term in the Hamiltonian, we find new terms such as Zeeman effects or spin-orbit interactions. In the same way, in the non-relativistic reduction of the scalar potential term in the Hamiltonian, we find the new additional potential. In fact, this new additional potential can reproduce the leap second of the earth revolution.

• Inertial Mass and Gravitational Mass :
From experiments, it is known that the inertial mass and gravitational mass are just the same. This equivalence of two masses is taken to be one of the grounds in constructing the general relativity. On the other hand, this equivalence is derived as a natural consequence in the new gravity model. This is one of the strong reasons why this new gravity model is a correct theory of gravity.

# Appendix C

# Planet Effects on Mercury Perihelion

In this Appendix, we discuss the Mercury perihelion shifts which should come from the gravitational interactions between Mercury and other planets such as Jupiter or Saturn. This calculation can be carried out in the perturbation theory of the Newton dynamics, which is rather new to the classical mechanics. Here, we should compare the numerical results with those calculated by Newcomb in 1898.

## C.1 Planet Effects on Mercury Perihelion

The motion of the other planets should affect on the Mercury orbits. However, this is the three body problems, and thus it is not easy to solve the equation of motion in an exact fashion. Here, we develop the perturbative treatment of the other planet motions. Suppose Mercury and the planet (Jupiter) are orbiting around the sun, and in this case, the Lagrangian can be written as

$$L = \frac{1}{2}m\dot{\boldsymbol{r}}^2 + \frac{GmM}{r} + \frac{1}{2}m_w\dot{\boldsymbol{r}_w}^2 + \frac{Gm_wM}{r_w} + \frac{Gmm_w}{|\boldsymbol{r}-\boldsymbol{r}_w|} \tag{C.1}$$

where $(m,\ \boldsymbol{r})$ and $(m_w,\ \boldsymbol{r}_w)$ denote the mass and coordinate of Mercury and the planet, respectively. The last term in the right side of eq.(C.1) is the gravitational potential between Mercury and the planet, and therefore, it should be much smaller than the gravitational force from the sun.

### C.1.1 The Same Plane of Planet Motions

**Here, we assume that the motion of Mercury and the planet must be in the same plane, and therefore we rewrite the Lagrangian in terms of polar coordinates in two dimensions**

$$\begin{aligned} L &= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) + \frac{GmM}{r} + \frac{1}{2}m_w(\dot{r_w}^2 + r_w^2\dot{\varphi_w}^2) + \frac{Gm_wM}{r_w} \\ &+ \frac{Gmm_w}{\sqrt{r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w)}}. \end{aligned} \tag{C.2}$$

**In this case, the Lagrange equation for Mercury can be written as**

$$m\ddot{r} = mr\dot{\varphi}^2 - \frac{GmM}{r^2} - \frac{Gmm_w(r - r_w\cos(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.3}$$

$$\frac{d}{dt}(mr^2\dot{\varphi}) = -\frac{GmMrr_w\sin(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.4}$$

$$m_w\ddot{r_w} = m_wr_w\dot{\varphi}^2 - \frac{Gm_wM}{r_w^2} - \frac{Gmm_w(r_w - r\cos(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.5}$$

$$\frac{d}{dt}(m_wr_w^2\dot{\varphi}) = -\frac{Gm_wMrr_w\sin(\varphi_w - \varphi))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}}. \tag{C.6}$$

### C.1.2 Motion of Mercury

**If we ignore the interaction between Mercury and the planet, then the Mercury orbit is just given as the Kepler problem, and the equations of motion become**

$$m\ddot{r} = mr\dot{\varphi}^2 - \frac{GmM}{r^2} \tag{C.7}$$

$$\frac{d}{dt}(mr^2\dot{\varphi}) = 0. \tag{C.8}$$

**Here, the solution of the orbit trajectory is given as**

$$r = \frac{A}{1 + \varepsilon\cos\varphi} \tag{C.9}$$

**where $A$ and $\varepsilon$ are written as**

$$A = \frac{\ell^2}{m\alpha}, \qquad \varepsilon = \sqrt{1 + \frac{2E\ell^2}{m\alpha^2}} \quad \textbf{with} \quad \alpha = GMm \tag{C.10}$$

**which should be taken as the unperturbed solution of the revolution orbit.**

## C.2 Approximate Estimation of Planet Effects

**Now we should make a perturbative calculation of the many body Kepler problem by assuming that the interaction between Mercury and the planet is sufficiently small. In this case, we can estimate the effects of other planets on the Mercury orbit. Here we write again the equation of motion for Mercury including the gravity from the other planet**

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} - \frac{Gm_w(r - r_w\cos(\varphi-\varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi-\varphi_w))^{\frac{3}{2}}}. \tag{C.11}$$

**Now we replace** $r,\ r_w$ **by the average orbit radius** $R,\ R_w$ **in the last term of the right side, and thus, the equation becomes**

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} - \frac{Gm_w(R - R_w\cos(\varphi-\varphi_w))}{(R^2 + R_w^2 - 2RR_w\cos(\varphi-\varphi_w))^{\frac{3}{2}}}. \tag{C.12}$$

**Below we present some approximate solution of eq.(C.12).**

### C.2.1 Legendre Expansion

**First we define the last term of eq.(C.12) by** $F$ **as**

$$F(x) \equiv -\frac{Gm_w(R - R_w x)}{(R^2 + R_w^2 - 2RR_w x)^{\frac{3}{2}}}, \qquad \textbf{with} \quad x = \cos(\varphi-\varphi_w) \tag{C.13}$$

**and we make the Legendre expansion**

$$F(x) = -\frac{Gm_w R}{(R^2 + R_w^2)^{\frac{3}{2}}} + \frac{Gm_w R_w(R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}}\,x + \cdots \ . \tag{C.14}$$

**Therefore we obtain the equation of motion**

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} + \frac{Gm_w R_w(R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}}\cos(\varphi-\varphi_w) \tag{C.15}$$

**where the constant term is irrelevant and thus we do not write it above.**

### C.2.2 Iteration Method

**Now we employ the iteration method in order to solve eq.(C.15). First we make use of the solution of the Kepler problem**

$$\varphi = \varphi^{(0)} + \omega t \quad (C.16)$$

$$\varphi_w = \varphi_w^{(0)} + \omega_w t \quad (C.17)$$

**and thus eq.(C.15) becomes**

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} + \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}} \cos(b + \beta t) \quad (C.18)$$

**where $b$ and $\beta$ should be given as**

$$b = \varphi^{(0)} - \varphi_w^{(0)}, \quad \beta = \omega - \omega_w. \quad (C.19)$$

### C.2.3 Particular Solution

**In order to solve eq.(C.18), we assume that the last term is sufficiently small and therefore $r$ may be written in the following shape as**

$$r = r^{(0)} + K \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}} \cos(b + \beta t) \quad (C.20)$$

**where $r^{(0)}$ denotes the Kepler solution of $r^{(0)} = \frac{A}{1+\varepsilon \cos\varphi}$. Now we insert the solution of eq.(C.20) into eq.(C.18), and we find the solution of $K$ as**

$$K = -\frac{1}{\beta^2}. \quad (C.21)$$

**Therefore, we obtain the approximate solution as**

$$r = r^{(0)} - \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}} \beta^2} \cos(b + \beta t). \quad (C.22)$$

## C.3 Effects of Other Planets on Mercury Perihelion

**Therefore we should put the Kepler solution for $r^{(0)}$ and thus the Mercury orbit can be written as**

$$\begin{aligned} r &= \frac{A}{1+\varepsilon\cos\varphi} - \frac{Gm_w R_w (R_w^2 - 2R^2)}{(R^2+R_w^2)^{\frac{5}{2}}\beta^2}\cos(b+\beta t) \\ &\simeq \frac{A}{1+\varepsilon\cos\varphi + \frac{Gm_w R_w (R_w^2-2R^2)}{R(R^2+R_w^2)^{\frac{5}{2}}(\omega-\omega_w)^2}\cos(b+\beta t)} \end{aligned} \tag{C.23}$$

**where we take $A \simeq R$ and also $\beta = \omega - \omega_w$. Here as for $\varepsilon_w$, we take**

$$\varepsilon_w \equiv \frac{Gm_w}{RR_w^2(\omega-\omega_w)^2}\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}} \tag{C.24}$$

**and using $b + \beta t = \varphi - \varphi_w$, we obtain**

$$r \simeq \frac{A}{1+\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w)}. \tag{C.25}$$

**This equation suggests that the Mercury perihelion may well be affected by the planet motions.**

### C.3.1 Numerical Evaluations

**Now we calculate the Mercury perihelion shifts due to the planet motions such as Jupiter or Venus. In order to do so, we first rewrite $\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w)$ terms as**

$$\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w) = c_1\cos\varphi + c_2\sin\varphi = \sqrt{c_1^2+c_2^2}\cos(\varphi+\delta) \tag{C.26}$$

**where $c_1$ and $c_2$ are defined as**

$$c_1 = \varepsilon + \varepsilon_w\cos\varphi_w \tag{C.27}$$

$$c_2 = \varepsilon_w\sin\varphi_w. \tag{C.28}$$

**Here $\cos\delta$ can be written as**

$$\cos\delta = \frac{c_1}{\sqrt{c_1^2+c_2^2}}. \tag{C.29}$$

**Further, $\varepsilon_w$ is much smaller than $\varepsilon$ and thus eq.(C.29) becomes**

$$\cos\delta = \frac{\varepsilon + \varepsilon_w \cos\varphi_w}{\sqrt{(\varepsilon + \varepsilon_w \cos\varphi_w)^2 + (\varepsilon_w \sin\varphi_w)^2}} \simeq 1 - \frac{1}{2}\left(\frac{\varepsilon_w}{\varepsilon}\right)^2 \sin^2\varphi_w. \tag{C.30}$$

## C.3.2 Average over One Period of Planet Motion

**Now we should make the average over one period of planet motion and therefore we find**

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2\varphi_w \, d\varphi_w = \frac{1}{2}. \tag{C.31}$$

**Thus, $\delta$ becomes**

$$\begin{aligned}\delta &\simeq \frac{\varepsilon_w}{\sqrt{2}\,\varepsilon} \simeq \frac{1}{\sqrt{2}\,\varepsilon}\frac{GM}{R_w^2}\frac{1}{R(\omega-\omega_w)^2}\left(\frac{m_w}{M}\right)\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}} \\ &\simeq \frac{R_w\,\omega_w^2}{\sqrt{2}\,\varepsilon R\,(\omega-\omega_w)^2}\left(\frac{m_w}{M}\right)\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}}\end{aligned} \tag{C.32}$$

**where the planet orbits are taken to be just the circle, for simplicity.**

### C.3.3 Numerical Results

**In order to calculate the effects of the planet motions on the $\delta$, we first write the properties of planets in Table 1. Here, numbers are shown in units of the earth.**

**Table 1**

| | Mercury | Venus | Mars | Jupiter | Saturn | Earth | Sun |
|---|---|---|---|---|---|---|---|
| Orbit Radius | 0.387 | 0.723 | 1.524 | 5.203 | 9.55 | 1.0 | |
| Mass | 0.055 | 0.815 | 0.107 | 317.8 | 95.2 | 1.0 | 332946.0 |
| Period | 0.241 | 0.615 | 1.881 | 11.86 | 29.5 | 1.0 | |
| $\omega$ | 4.15 | 1.626 | 0.532 | 0.0843 | 0.0339 | 1.0 | |

**In Table 2, we present the calculations of the values $\delta$ for one hundred years of averaging and the calculations are compared with the calculated results by Newcomb.**

**Table 2 The values of $\delta$ for one hundred years**

| Planets | Venus | Earth | Mars | Jupiter | Saturn | Sum of Planets |
|---|---|---|---|---|---|---|
| $\delta$ by eq.(C.32) | 49.7 | 27.4 | 0.77 | 32.1 | 1.14 | 111.1 |
| $\delta$ by Newcomb | 56.8 | 18.8 | 0.51 | 31.7 | 1.5 | 109.3 |

**As one sees, the agreement between the present calculation and Newcomb results is surprisingly good [24]. Here we do not verify the calculation of Newcomb for the other planet effects on the Mercury perihelion shifts, and instead we simply employ his calculated results.**

### C.3.4 Comparison with Experiments

**The observed values of the Mercury perihelion shifts are often quoted in some of the old textbooks. However, it should be very difficult to find some reliable numbers of the Mercury perihelion shifts since these values are determined for 100 years of observation period in 19 century. In this respect, the comparison between the calculation and observation should be a homework problem for readers.**

# Appendix D

# No Time Delay in Moving Frame

**From the Lorentz transformation of eq.(1.1), it looks that time in the moving frame deviates from the rest frame. However, $t$ and $x$ are variables, and thus, they are not directly related to physical observables. Below we examine whether the time difference of $\Delta t$ in the Gedanken experiment should be delayed or not.**

## D.1 Incorrect Gedanken Experiment

**Here we first explain the time difference $\Delta t$ in the Gedanken experiment which is often discussed in the science history, though it is incorrect. First, we consider a train (moving frame) which is driving in the straight line with a constant velocity $v$. We assume that there should be big mirror wall in parallel to the straight line with its distance of $\ell$.**

### D.1.1 Time Difference of Moving Frame from Rest Frame

**First, an observer in the train emits laser beams against mirror wall. In this case, the observer in the train should not notice that the train is moving. Now this observer should detect the reflected laser beam and should measure the time difference ($2\Delta\tau$). In this case, we see**

$$\ell = c\Delta\tau. \tag{D.1}$$

**On the other hand, an observer at the rest frame should detect the laser beam which reflects and travels through the triangle trajectory. In this case, the time difference ($2\Delta t$) should be**

$$\sqrt{(c\Delta t)^2 - \ell^2} = v\Delta t. \tag{D.2}$$

Therefore, we find

$$\sqrt{c^2 - v^2}\ \Delta t = c\Delta\tau \tag{D.3}$$

which gives us the following relation between the time differences of $\Delta\tau$ and $\Delta t$ as

$$\Delta\tau = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta t. \tag{D.4}$$

This suggests that the time difference in the moving frame seems to be somewhat smaller than that of the rest frame.

### D.1.2 Time Difference of Rest Frame from Moving Frame

Now we should carry out the same type of Gedanken experiment from the observer at the moving frame. In this case, the rest frame is moving with the velocity of $-v$ for the observer of the moving frame. This can be easily seen if we solve the Lorentz transformation the other way around

$$x' = \gamma(x - vt), \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right), \qquad y' = y, \qquad z' = z. \tag{D.5}$$

Here we see that the rest frame is moving with its velocity of $(-v)$. But otherwise, everything is just the same as in the previous case. In this case, the observer in the rest frame emits laser beams against mirror wall, and the observer in the train should detect the reflected laser beam and should measure the time difference $(2\Delta ct)$. Thus, we find

$$\Delta t = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta\tau. \tag{D.6}$$

### D.1.3 Inconsistency of Time Difference

What is going on? The results of eqs. (D.4) and (D.6) contradict with each other. Since $\Delta t$ and $\Delta\tau$ should be observables in the Gedanken experiment, there must be something wrong there.

## D.2 Where is Incorrect Process in Gedanken Experiment?

What should be incorrect inductions in the Gedanken experiment? This can be easily seen if we look into eq. (D.2). After $\Delta t$, we took the coordinate of the train as $\Delta x' = \Delta x + v\Delta t$, which is wrong. The correct coordinate after $\Delta t$ should be given by the Lorentz transformation as

$$\Delta x' = \gamma v \Delta t. \tag{D.7}$$

Thus, we should replace in the following way

$$v\Delta t \Longrightarrow \gamma v \Delta t, \qquad c\Delta t \Longrightarrow \gamma c \Delta t. \tag{D.8}$$

Therefore, eq. (D.4) becomes

$$\begin{aligned} \Delta \tau &= \sqrt{1 - \frac{v^2}{c^2}} \times \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta t \\ &= \Delta t. \end{aligned}$$

This clearly shows that there is no time delay, and there is no inconsistency. This is just all what we see from the relativity.

### D.2.1 No Time Delay in Moving Frame!

From the Gedanken experiment, we see that there is no time delay in the moving frame as compared to the rest frame. This is quite reasonable since the relativity only states that any inertial frames should produce the same results of all physical observables.

In fact, the time interval is defined from the earth period $T$ around the sun. It is, of course, clear that, in any inertial system, the period $T$ is the same. Therefore, there is no time delay in any inertial system even if it is moving very fast.

## D.3 Examples of Relativity

Here we should discuss possible observables when two inertial frames are involved in physical processes. It should be noted that this consideration is only related to the kinematics, and therefore, we cannot learn anything about dynamics of physical processes.

### D.3.1 Doppler Effect of Light

When a star is moving away from the earth, then lights emitted from this star should be affected by the Lorentz transformation, and this is known as the Doppler effect. Let consider that a star is going away with its velocity $v$. The momentum $p$ of light emitted at the star should become $p'$ on the earth, and this relation is given by the Lorentz transformation as

$$p' = \gamma\left(p - \frac{vE}{c^2}\right) = \gamma\left(p - \frac{vp}{c}\right) = \frac{p\left(1-\frac{v}{c}\right)}{\sqrt{(1-\frac{v^2}{c^2})}} = p\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}. \tag{D.9}$$

This shows that the momentum of light is decreased. If we express the above relation in terms of wave length, then we obtain

$$\lambda' = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,\lambda. \tag{D.10}$$

Since the wave length of the observed light becomes longer, we call it “red shift”. It should be noted that this naming has no physical meaning. It simply says that red light has a longer wave length than that of blue light. The physical reason of the Doppler shift is simply because the energy and momentum make a four dimensional vector and therefore this is affected by the Lorentz transformation.

### D.3.2 Life Time of Muon Produced in Atmosphere

High energy cosmic ray (protons) may collide with atmospheric $N_2$ or other molecule and may produce muons with the mass of $m_\mu = 105.6$ MeV/$c^2$. The life time $\tau_0$ of this lepton is around $\tau_0 \simeq 2\times 10^{-6}$ s. Therefore, muon is unstable. Now a question is as to whether the life time of muon may be affected by the Lorentz transformation or not. This problem is often discussed in science history, but here we should present a right description of muon as to how far it can travel in the air.

**Now the life time $\tau_0$ can be written in terms of decay width $\Gamma$ as**

$$\tau_0 = \frac{\hbar}{\Gamma}. \tag{D.11}$$

**Here we note that $\Gamma$ is a Lorentz invariant quantity. Therefore, the life time is also Lorentz invariant, and thus the life time of muon should be the same in any inertial frame.**

### D.3.3 Travel Distance $L$ of Muon

**Now we should calculate the travel distance $L$ of muon after it is created from the collision of protons with atmosphere. This can be evaluated from the Lorentz transformation $x = \gamma(x' + vt')$ as**

$$L = \gamma v \tau_0. \tag{D.12}$$

**Here we take, as an example, muon with its energy of 1 GeV. In this case, the velocity of muon can be approximated by light velocity of $c$. The Lorentz factor $\gamma$ should be $\gamma \simeq 10.6$. Therefore, the value of $L$ becomes**

$$L = \gamma v \tau_0 = 10.6 \times 3 \times 10^8 \times 2 \times 10^{-6} \simeq 6.3 \text{ km} \tag{D.13}$$

**which is longer by $\gamma$ than $v\tau_0$. This indicates that the muon produced in the atmosphere may well have some chance to be observed on the earth.**

### D.3.4 Accelerator Experiment

**Unstable particles created by the large accelerator should travel the distance which is given by eq. (D.12). This is longer by a factor of $\gamma$ than $v\tau_0$, but it has nothing to do with the delay of life time of unstable particles. It is simply due to the Lorentz transformation.**

# Appendix E

# New Evaluation of Rayleigh Scattering

**Here we describe the theory of Rayleigh scattering in terms of quantum mechanics terminology. First, we briefly review the cross section of Rayleigh scattering which is obtained by the classical electrodynamics. However, it is shown that the cross section commonly used until now is ten orders of magnitude smaller than the cross section which is calculated by Compton scattering evaluation. Therefore, there must be something wrong with the old fashioned Rayleigh scattering evaluation.**

## E.1 Interaction of Photon with Electron

**Photon should interact only with electron, and thus photon should scatter with electrons in atoms. The interaction Hamiltonian can be written as**

$$H' = -\frac{e}{m}\boldsymbol{p}\cdot\boldsymbol{A}(x). \tag{E.1}$$

**This expression is non-relativistic, but it is just the same as the relativistic case, apart from the spin part which is not included here. In this sense, this interaction of eq.(E.1) must be sufficient as long as we treat the interaction of photon with atomic electrons. Here, $m$ denotes the mass of electron, and $\boldsymbol{p}$ is a momentum operator of electron. Also, $\boldsymbol{A}(x)$ denotes the vector potential which is given as**

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k},\lambda} \frac{\boldsymbol{\epsilon}_k^\lambda}{\sqrt{2\omega_k V}} \left( c_{\boldsymbol{k},\lambda}^\dagger e^{ikx} + c_{\boldsymbol{k},\lambda} e^{-ikx} \right) \tag{E.2}$$

**where $kx \equiv \omega_k t - \boldsymbol{k}\cdot\boldsymbol{r}$. Also, $c^\dagger_{\boldsymbol{k},\lambda}$ ans $c_{\boldsymbol{k},\lambda}$ should be the creation and annihilation operators of photon.**

### E.1.1 Scattering T-Matrix in Second Order Perturbation

**We consider the scattering of photon with electrons in atoms. In this case, the scattering can be described in terms of the second order perturbation theory. The interaction Hamiltonian is given in eq.(E.1). The elastic scattering T-matrix of photon with electron in atom can be written as**

$$\begin{aligned} T &= \sum_n \langle\phi_0|H'|\phi_n\rangle\langle\phi_n|H'|\phi_0\rangle \left(\frac{1}{E_n - E_i - k + i\varepsilon} + \frac{1}{E_n - E_i + k + i\varepsilon}\right) \\ &\simeq \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_{n,\lambda} \frac{2\langle\phi_0|(i\boldsymbol{k}\cdot\boldsymbol{r})(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_n\rangle\langle\phi_n|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(-i\boldsymbol{k}\cdot\boldsymbol{r})|\phi_0\rangle}{E_n - E_i} \end{aligned}$$

**where the initial state of the atom can be written as $|i\rangle = |\phi_0(\boldsymbol{r})\rangle$. Here, $\phi_0(\boldsymbol{r})$ denotes the ground state of electron in the atom. Now, we assume that $E_n - E_i \gg k$ which should be well satisfied in this discussion. Also, $|\phi_n\rangle$ denotes the $n-$th excited state of the atom. $E_i$ and $E_n$ denote the eigenvalues of the ground state and the $n-$th excited state in the atom, respectively. Here the photon state is approximated as**

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \simeq 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots \tag{E.3}$$

**which is the long wave length approximation. Note that the wave length of visible lights must be $k \simeq 1.2 \times 10^5$ cm while the radius of stom should be $r \leq 1.0 \times 10^{-7}$ cm$^{-1}$. Thus, we find $kr \simeq 10^{-2}$ which is sufficiently small for the expansion in eq.(E.3).**

### E.1.2 Evaluation of Scattering T-Matrix

**Here, we make the closure approximation, and assume**

$$\Delta E \equiv E_n - E_i$$

**where the $n$ dependence is neglected in $\Delta E$. In this case, we find**

$$T = \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_\lambda \frac{2\langle\phi_0|(\boldsymbol{k}\cdot\boldsymbol{r})(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(\boldsymbol{k}\cdot\boldsymbol{r})|\phi_0\rangle}{\Delta E} \simeq \left(\frac{e^2}{Vkm^2}\right)\frac{k^2}{\Delta E}.$$

## E.2 Cross Section of Rayleigh Scattering

**We should evaluate the cross section of Rayleigh scattering and estimate numerically the order of magnitude of the cross section. The differential cross section can be written in terms of the T-matrix as**

$$\frac{d\sigma}{d\Omega} = 2\pi|T|^2\frac{V}{(2\pi)^3}k^2\left(\frac{V}{c}\right) = \frac{4\alpha^2k^4}{m^4(\Delta E)^2} = \frac{r_0^2}{2}\left(\frac{\lambda_0}{\lambda}\right)^4 \tag{E.4}$$

**where we introduce**

$$\lambda = \frac{2\pi}{k}, \qquad r_0 = \frac{\alpha}{m} = 2.82 \times 10^{-13} \quad \text{cm.} \tag{E.5}$$

**Also, we define** $\lambda_0$

$$\lambda_0^4 \equiv \frac{8(2\pi)^4}{m^2(\Delta E)^2}. \tag{E.6}$$

**As one sees, eq.(E.4) is just the cross section of Rayleigh scattering.**

### E.2.1 Numerical Value of $\lambda_0$

**Now we should make a rough estimation of** $\lambda_0$ **value. Here, we take** $m = 0.51$ **MeV/c$^2$ and** $\Delta E \simeq 7$ **eV. In this case, we find**

$$\lambda_0 \simeq 1.1 \times 10^{-7} \text{ cm.} \tag{E.7}$$

**For visible lights, we see** $\lambda \simeq 4.5 \times 10^{-5}$ cm **and thus**

$$\left(\frac{\lambda_0}{\lambda}\right)^4 \simeq 3.6 \times 10^{-11} \tag{E.8}$$

**which is extremely small. Thus, the Rayleigh scattering cross section becomes**

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ray} \simeq 3.6 \times 10^{-11} \times \left(\frac{r_0^2}{2}\right). \tag{E.9}$$

**This indicates that the Rayleigh scattering cannot be applied to nature.**

## E.3 Atomic Compton Scattering

Here we should present the calculation of the differential cross section of atomic Compton scattering where photon scatters with atomic electrons. In this case, it should involve many body effects in the scattering process as a result.

### E.3.1 Evaluation of Scattering T-Matrix

The scattering T-matrix between photon and electrons in atoms can be calculated from the second order perturbation theory as

$$\begin{aligned} T_{A-Comp} &= \sum_n \langle\phi_0|H'|\phi_n\rangle\langle\phi_n|H'|\phi_0\rangle \left(\frac{1}{E_n - E_i - k + i\varepsilon} + \frac{1}{E_n - E_i + k + i\varepsilon}\right) \\ &= \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_{n,\lambda} \langle\phi_0|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_n\rangle\langle\phi_n|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_0\rangle \frac{2(E_n - E_i)}{(E_n - E_i)^2 - k^2} \end{aligned} \quad \text{(E.10)}$$

which should generate the biggest contribution to the cross section of atomic Compton scattering. Here, we ignore the pole contribution, and further we make the long wave length approximation. In this case, the wave function of photon can be expanded as

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots \quad \text{(E.11)}$$

where we take only the first term in eq.(E.11). This approximation can be justified since we consider the scattering of visible lights with electrons.

The similar calculation was carried out in the textbook of Sakurai [25]. However, the treatment in this textbook contains the effects from the $\boldsymbol{A}^2$ term which should not be included. Further, the approximations employed there are quite rough and therefore the result of the cross section cannot be reliable, even though the textbook claimed that the shape of Rayleigh scattering cross section could be reproduced.

### E.3.2 Closure Approximation and Virial Theorem

Now we rewrite the T-matrix of eq.(E.10) by making use of closure approximation and obtain

$$\begin{aligned} T_{A-Comp} &= \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \frac{2}{\Delta E} \sum_\lambda \langle\phi_0|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)^2|\phi_0\rangle F_k \\ &= \left(\frac{e^2}{2Vkm}\right)\left(\frac{4}{3\Delta E}\right)\left\langle\phi_0\left|\frac{\boldsymbol{p}^2}{2m}\right|\phi_0\right\rangle F_k \end{aligned} \quad \text{(E.12)}$$

**where $F_k$ is defined as**

$$F_k \equiv \frac{1}{1-\left(\frac{k}{\Delta E}\right)^2}. \tag{E.13}$$

**Further, by making use of the Virial theorem for the Coulomb potential of $[V_c(r) = -\frac{Ze^2}{r}]$, we find**

$$\langle \phi_0 \left| \frac{\boldsymbol{p}^2}{2m} \right| \phi_0 \rangle = -\frac{1}{2}\langle \phi_0 \left| V_c(r) \right| \phi_0 \rangle = |E_0|. \tag{E.14}$$

**Here $E_0$ denotes the eigenvalue of the ground state in atom. As an average value of $\Delta E$, we take**

$$\Delta E \simeq \frac{4}{3}|E_0|. \tag{E.15}$$

**This is an approximation, but it should be rather reliable for the estimation of atomic excitations. Thus, we can write $T_{Comp}$**

$$T_{A-Comp} = \frac{e^2}{2Vkm} F_k.$$

## E.3.3 Cross Section of Atomic Compton Scattering

**Now, the differential cross section of atomic Compton scattering can be written as**

$$\left(\frac{d\sigma}{d\Omega}\right)_{A-Comp} = 2\pi \left(\frac{e^2}{2Vkm}\right)^2 \frac{V^2 k^2}{(2\pi)^3} |F_k|^2 = r_0^2 |F_k|^2. \tag{E.16}$$

**In the case of visible lights, we may take**

$$|F_k|^2 \simeq 1. \tag{E.17}$$

**Therefore, we see that the differential cross section of atomic Compton scattering should be almost the same as the normal Compton scattering cross section of photon with electrons. Therefore, it is clear that the atomic Compton scattering is much larger than the Rayleigh scattering for visible lights.**

### E.3.4 Comparison of Atomic Compton Scattering and Rayleigh Scattering

**Here we should compare two cross sections between atomic Compton scattering and Rayleigh scattering. It is easy to see where the difference between two scattering processes emerges. In order to see it, we write again the expansion of the photon wave function**

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \simeq 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots . \tag{E.18}$$

**If we take the first term in eq.(E.18), then this corresponds to the atomic Compton scattering process. On the other hand, the second term of eq.(E.18) corresponds to the Rayleigh scattering process. In fact, the magnitude of ($\boldsymbol{k}\cdot\boldsymbol{r}$) should be much smaller than 1 for visible lights.**

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