Physics of Leap Second

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1. Newcomb Time and Atomic Watch

Orbital Period of Earth Revolution : Average Regression Year T_0.

$T_0 = 365.242189$ **(day)** $= 3.155692513 \times 10^7$ **(s)**

This defines the Newtonian time.

- **Newcomb time : Orbital Period of Earth Revolution.**
  Newcomb defines a second by Newtonian Orbital Period.

- **Time of Atomic Watch : Orbital Period of Earth Revolution.**
  Measured value by Atomic Watch deviates 0.61 second (Leap Second) from Newtonian Orbital Period.

2. Origin of Leap Second

What is leap second ? :
The effect of the additional gravitational force
on Orbital Period of Earth Revolution

The observed period T is longer then T_0 by $\Delta T = 0.621$ (s)
$T = T_0 + \Delta T$

(1) Newtonian Orbital Period of Earth Revolution

Orbital Period From perihelion to perihelion T_0 (s)

(2) Orbital Period measured by Atomic Watch

From perihelion to perihelion : 0.62 (s) longer than T_0

• **Additional Gravitational Force : Relativistic Effect**

Relativistic Effect on Orbital Motion : $\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8}$
Leap Second $\sim 2 \times 10^{-8}$ **: Good agreement**

• **Additional Gravitational Force : Prediction** $\boxed{\Delta T_{(Pred.)} = 0.621(s)}$

Reference :

(1) "Symmetry and Its Breaking in Quantum Field Theory"
(T. Fujita, Nova Science Publishers, 2011, 2nd edition)

(2) "Fundamental Problems in Quantum Field Theory"
(T. Fujita and N. Kanda, Bentham Publishers, 2013)

3. Earth's Rotation and Tidal Force

[Wrong Theory] : Tidal force may affect on Earth's Rotation ?

[Wrong claim] : Tidal force may push up matters to the surface ?
Moment of Inertia may become larger ?
Thus, rotation velocity may become slower ?

• **The reasons why this theory is incorrect**

(1) Tidal Force is conservative and thus does not make Work !
Tidal Force cannot change matter distribution in Earth.

• **Basic Mechanics : Conservative Force and Work**

$\boldsymbol{F} = -\boldsymbol{\nabla} V(r)$ is called conservative force. Or $\boldsymbol{\nabla} \times \boldsymbol{F} = 0$.
The gravity is a conservative force. The Work W is
$W(A \to B \to A) = \oint \boldsymbol{F} \cdot d\boldsymbol{r} = -\left[V(r)\right]_{A-B-A} = 0$
and thus does not make any Work.

(2) Matter in Earth cannot move to the surface
since this is against the gravity of Earth.

• **Period of Earth's Rotation never changes !**

4. Work of Non-conservative Force

- **Work of non-conservative force :** **we consider non-conservative force ($\boldsymbol{\nabla} \times \boldsymbol{F} \neq 0$)**

  $$\boxed{\boldsymbol{F} = (-kx + \varepsilon y)\boldsymbol{e}_x - ky\boldsymbol{e}_y}$$

  $F_z = 0$ has no loss of generality.

  When $\varepsilon = 0$, it is a conservative force ($\boldsymbol{\nabla} \times \boldsymbol{F} = 0$).

- **Definition of Work W :** $W = \oint \boldsymbol{F} \cdot d\boldsymbol{r}$

  The motion of a particle is $\boxed{x = a\cos\omega t,\ \ y = a\sin\omega t}$**.**

  In this case, W is calculated with its period T

  $$\boxed{W = \oint \boldsymbol{F} \cdot d\boldsymbol{r} = \int_0^T (-kx\dot{x} + \varepsilon y\dot{x} - ky\dot{y})dt}$$

  where $\omega T = 2\pi$. Thus

  $$\boxed{W = -\varepsilon a^2\omega \int_0^T (\sin\omega t)^2 dt = -\pi\varepsilon a^2}.$$

Therefore, the conservative force cannot make Work.
But the non-conservative force does make Work $\Rightarrow$
Thus, Energy must be consumed.

5. Leap Second : Prediction of General Relativity

Correction of General Relativity to Newton Mechanics

- **Correction Term :** **Correction Potential ΔV_{GR}**

  $\Delta V_{GR} \simeq -\frac{3}{mc^2}\left(\frac{GmM}{r}\right)^2$: **This is attractive !**

  (If M is very large, the gravitational collapse occurs.)

- **Effect on Leap Second :**

  Attractive Force : Area of elliptical orbit becomes smaller.

  Correction to Orbital Period : Shorter $\Rightarrow$ Anti-leap second

  (Orbital Period T and Area S are related as $S = \frac{\ell}{2m}T$
  where $\ell,\ m$ is angular momentum and mass of Earth.)

- **Correction of General Relativity : disagree with observation !**

6. Mercury Perihelion Shifts : Problem of General Relativity Prediction

Mercury Perihelion Shift : $42''$ per 100 years

- **Problem of Theoretical Calculation :**

  (1) Mercury Orbit Change due to Additional Potential

  (2) Mercury Orbit gets larger, Orbit Period longer

  (3) Orbit Change affects much larger on Perihelion Shift than θ dependence of Elliptic Orbit

  (4) General Relativity : Orbit Size smaller, Period shorter

- **Observed Perihelion Shift :** $\delta\theta_{Obs} \simeq 7.8 \times 10^{-8}$

  New Gravity : $\delta\theta_{Th} \simeq 4.8 \times 10^{-8}$

  General Relativity : $\delta\theta_{GR} \simeq 3.3 \times 10^{-8}$ **(No Orbit Change)**

  : $\boxed{\delta\theta_{GR} \simeq -30 \times 10^{-8}}$ **(Orbit Change)**

- **Perihelion Shift : May not be Physical Observables.**
- **Physical Observables : Orbit Period (No ambiguity)**