Physics of CP Violation

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Preface

In this short note, I should discuss a new but phenomenological approach to understand the physics of CP-violation and/or T-violation. This must be somewhat similar to the physics of Ohm's law where the time reversal invariance is violated. In the case of Ohm's law, T-violation is induced when the conduction electron should lose its energy from the collision with electrons of lattice, and therefore its flux should be reduced, and this is related to the apparent T-violation of Ohm's law.

Here, I should present a qualitative picture of CP-violation in the decay of neutral meson K_L^0 since this CP-violation may well be induced by the flux loss of K_L^0 current in the decay process. However, I do not present any realistic calculations of CP-violation of K_L^0 system, and the main part of calculations should be left for young physicists as some research works. In this respect, only a general picture of CP-violation is explained in this short note.

I hope that this note may help young people understand some basic physics in depth.

Contents

Chapter 1

CP and T Violation

In field theory, the CPT theorem must hold rigorously where C denotes the charge conjugation, P parity transformation and T time reversal transformation, respectively. Therefore, if CP invariance is violated (CP-violation), then this means that the time reversal invariance should also be violated (T-violation) as well.

1.1 CPT Theorem

In quantum field theory, the CPT theorem holds rigorously, and this can be well explained in field theory textbooks [1, 2]. This means that if we make the C, P and T transformation successively, then the Lagrangian density of field theory must be invariant.

1.2 T-violation

Until now, there is no fundamental interaction which should violate the time reversal invariance, apart from the CP-violation of K_L^0 decay in the weak interaction process. In spite of hard works, no evidence of T-violation has been observed in the neutron EDM measurements.

In this respect, the CP-violation observed in the decay of neutral meson K_L^0 must be quite special in modern field theory. Therefore, it should be important to clarify the essence of T-violation physics as to whether the T-violation should be induced by the fundamental interaction or something else like Ohm's law.

1.3 Apparent T-violation in Ohm's Law

As we discuss later, Ohm's law violates apparently the time reversal invariance, but this is originated from the loss of conduction electron flux. Therefore, the T-violation of Ohm's law has nothing to do with the symmetry in the fundamental interactions. The main reason of apparent T-violation of Ohm's law may well be the result of many body scattering of conduction electrons with lattice within electromagnetic interactions. Unfortunately, however, we do not fully understand the basic mechanism of Ohm's law in terms of microscopic calculations at the present stage.

1.3.1 Physical Process in Flow of Time

The apparent T-violation in Ohm's law may well be connected to the physical process in the time flow. In Ohm's law, the conduction electrons should lose their flux and energy in the time flow, and this cannot be reversed. Also, we should note that the physical process of Ohm's law is not in the stationary state, but rather it is in the scattering state of conduction electrons. This may well be the main reason of the T-violation in Ohm's law.

1.4 Neutron EDM

The most stringent experimental test of T-violation must come from the neutron electric dipole moments (EDM) whose existence should present a direct evidence of the T-violation term. The EDM of neutron should be in the stationary state, and therefore, the finite value of EDM of neutron must correspond to the real T-violation.

The Lagrangian density corresponding to the EDM of neutron can be written as

$$\mathcal{L}_{edm} = \frac{i}{2} d_n \bar{\psi} \sigma_{\mu\nu} \gamma^5 \psi F^{\mu\nu} \tag{1.1}$$

where d_n denotes the EDM of neutron. The Hamiltonian of the above EDM in the non-relativistic limit can be written as

$$H_{edm} \simeq d_n \boldsymbol{\sigma} \cdot \boldsymbol{E}. \tag{1.2}$$

Until now, a finite value of neutron EDM has not been observed yet.

1.4.1 Experimental Limit of Neutron EDM

The latest best limit of neutron EDM is

$$d_n \leq (0.0 \pm 1.1) \times 10^{-26} \text{ e} \cdot \text{cm}. \tag{1.3}$$

Therefore, at the present stage, there is no evidence of T-violation from the neutron EDM measurement [3].

Chapter 2

Experimental Evidence of CP Violation

The CP violation should be related to the observed rare decay of neutral meson K_L^0. The eigenvalue of CP operator of K_L^0 becomes

$$CP|K_L^0\rangle = -|K_L^0\rangle. \tag{2.1}$$

This means that the eigenvalue of CP for neutral meson K_L^0 should be (-1). However, people observed the decay process of

$$K_L^0 \to \pi^+ + \pi^- \tag{2.2}$$

where

$$CP|\pi^+\pi^-\rangle = |\pi^+\pi^-\rangle. \tag{2.3}$$

Therefore, the $\pi^+\pi^-$ system should have a different CP eigenvalue from the K_L^0, and thus, this decay process should violate the CP invariance [4].

Chapter 3

Ohm's Law

In electromagnetisms, there is an important law which is called Ohm's law. This law can be written as

$$\boldsymbol{j} = \kappa \boldsymbol{E} \tag{3.1}$$

where $\boldsymbol{j}$ and $\boldsymbol{E}$ denote the electric current and electric field, respectively. κ is called conductivity.

3.1 T-violation of Ohm's Law

It is simple to check that Ohm's law should violate the T-invariance. This can be easily seen since, under the time reversal transformation of $t \to -t$, we find

$$\begin{aligned} \boldsymbol{j} &\to -\boldsymbol{j} \\ \boldsymbol{E} &\to \boldsymbol{E}. \end{aligned} \tag{3.2}$$

Therefore, eq.(3.1) should not hold under the T-transformation. This means that Ohm's law cannot be taken as the fundamental equation of motion. In this case, we should understand the basic mechanism of Ohm's law in terms of scattering process between conduction electrons and lattice, even though it should be extremely difficult to do so.

3.2 Mechanism of Ohm's Law and Origin of T-violation

Now, a question may arise as to why Ohm's law violates apparently the T-invariance. Here, we should study the basic physics of Ohm's law itself, though qualitatively.

The mechanism of Ohm's law must be connected to the scattering process of conduction electrons with electrons of lattice. In this case, the conduction electron should lose its flux and energy during the collisions with lattice electrons. Until now, there is no reliable calculation of the scattering process since it should be quite involved.

3.2.1 T-Violation of Ohm's Law

Once the conduction electrons lose their flux and energy, then this physical process must be time-irreversible. This is clear since the process cannot be reversed. Therefore, this process must be related to the apparent T-violation, though further studies should be needed.

Chapter 4

Physics of CP Violation

Here, we discuss a possible scenario of understanding the CP violation in terms of some disappearance of kaon currents in the kaon production experiments.

4.1 Eigenstate of CP

Two meson states K^0 and $\bar{K}^0$ are not the eigenstate of CP operator, namely

$$CP|K^0\rangle = |\bar{K}^0\rangle$$
$$CP|\bar{K}^0\rangle = |K^0\rangle. \tag{4.1}$$

In this case, the eigenstate of CP operator for two meson states can be made as

$$|K_a\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$
$$|K_b\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle). \tag{4.2}$$

Therefore, these two states should be observables. In fact, the states K_L^0 and K_S^0 are observed in the decay experiments.

4.2 CP Violation in K_L^0 Decay

The decay of K_L^0 meson into two pions

$$K_L^0 \to \pi^+ + \pi^- \tag{4.3}$$

are observed in decay experiments. The observation of this process should indicate the CP-violation and thus the T-violation as well.

4.2.1 Origin of T-violation in K_L^0 Decay

A question may arize as to whether this T-violation should be caused by some fundamental interactions or not. Until now, it is believed that the fundamental interaction should violate time reversal invariance, and thus the CP violation of K_L^0 meson decay can be understood in terms of the complex phase of fundamental interactions as proposed by Kobayashi-Maskawa [5].

4.3 Alternative Picture of T-violation in K_L^0 Decay

However, there may well be some possibility that the CP violation of K_L^0 decay should be connected to the loss of flux of K_L^0 decay process. In this case, the K_L^0 decay process may well violate apparently the T-invariance. This is clear since the decay process is irreversible and thus some part of flux K_L^0 may be lost. The basic and important question should be as to how the T-violation of the K_L^0 decay process can be transformed into the CP-violation.

4.3.1 Home Work Problem

Here, however, we do not discuss it further, and this problem should be left for young physicists as Ph.D thesis or research works though it may require some hard works. The main point should be to clarify the connection of T-violation with the CP-violation in the decay process, and this should be worked out in some way or another.

Chapter 5

Quantum Field Theory

By now, quantum field theory is well established since there is no divergence in any physical observables. This includes the field theoretical construction of gravity, and the field theory of gravity should describe all the physical observables related to gravity [6]. In addition, there is a rigorous proof that the general relativity has nothing to do with physics [7]. Therefore, the basic problem of nature is well described by quantum field theory, and all the difficulties of describing nature should be originated from the many body nature.

Here, we briefly write down the Lagrangian densities of four interactions (QED, weak interactions, gravity and QCD) since the Lagrangian density should be the starting point of describing nature.

5.1 Quantum Electrodynamics and Gravity

The Lagrangian density of quantum electrodynamics (QED) and gravity can be written as

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - e\bar{\psi}\gamma^{\mu}A_{\mu}\psi - m(1+g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\upsilon\mu\nu} + \frac{1}{2}\partial_{\mu}\mathcal{G}\ \partial^{\mu}\mathcal{G} \tag{5.1}$$

where A_{μ} and $\mathcal{G}$ denote the vector potential and gravitational field, respectively. Here, $F^{\mu\nu}$ denotes the field strength and is defined as

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}. \tag{5.2}$$

5.2 Weak Interactions

We should write the simplest Lagrangian density for two flavor leptons which couple to the SU(2) vector fields W_μ^a

$$\mathcal{L} = \bar{\Psi}_\ell(i\partial_\mu\gamma^\mu - m)\Psi_\ell - gJ_\mu^a W^{\mu,a} + \frac{1}{2}M^2 W_\mu^a W^{\mu,a} - \frac{1}{4}G_{\mu\nu}^a G^{\mu\nu,a} \tag{5.3}$$

where M denotes the mass of the vector boson, and the weal vector field W_μ^a can be written as

$$W_\mu^a = (W_\mu^+, Z^0, W_\mu^-). \tag{5.4}$$

The lepton wave function Ψ_ℓ has two components

$$\Psi_\ell = \begin{pmatrix} \psi_e \\ \psi_\nu \end{pmatrix}. \tag{5.5}$$

Correspondingly, the mass matrix can be written as

$$m = \begin{pmatrix} m_e & 0 \\ 0 & m_\nu \end{pmatrix}. \tag{5.6}$$

The fermion current J_μ^a and the field strength $G_{\mu\nu}^a$ are defined as

$$J_\mu^a = \bar{\Psi}_\ell\gamma_\mu(1-\gamma_5)\tau^a\Psi_\ell, \qquad G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a. \tag{5.7}$$

The Lagrangian density of eq.(5.3) is almost the same as the standard model Lagrangian density, apart from the Higgs fields and the abelian nature. In fact, there is no experiment in weak process which cannot be described by the Lagrangian density of eq.(5.3).

5.3 Lagrangian Density of QCD

The Lagrangian density of quantum chromodynamics (QCD) for quark fields ψ with $SU(N_c)$ colors is written as [8]

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m_0)\psi - \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} \tag{5.8}$$

where $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]. \tag{5.9}$$

Here, the gluon field A_μ is given as

$$A_\mu = A_\mu^a T^a \equiv \sum_{a=1}^{N_c^2-1} A_\mu^a T^a \tag{5.10}$$

where T^a corresponds to the generator of $SU(N_c)$ group and satisfies the following commutation relations

$$[T^a, T^b] = iC^{abc}T^c. \tag{5.11}$$

C^{abc} denotes the structure constant of group generators. For $SU(2)$ case, the structure constant C^{abc} becomes just the anti-symmetric symbol ϵ_{abc}. In eq.(5.8), Tr { } means the trace of the group generators of $SU(N_c)$, and the generators T^a are normalized according to

$$\mathrm{Tr}\{T^aT^b\} = \frac{1}{2}\delta^{ab}. \tag{5.12}$$

This Lagrangian density is invariant under the following gauge transformation

$$\psi' = (1 - ig\chi)\psi = (1 - igT^a\chi^a)\psi, \quad \text{with} \quad \chi = T^a\chi^a \tag{5.13}$$
$$A'^a_\mu = A^a_\mu - gC^{abc}A^b_\mu\chi^c + \partial_\mu\chi^a \tag{5.14}$$

where χ depends on space and time as $\chi = \chi(t, \boldsymbol{r})$ which is infinitesimally small.

5.3.1 Color Charge of Quarks

The Lagrangian density of QCD [eq.(5.8)] is invariant under the local gauge transformation. However, the color charges for quark state [ψ] and gluon

state $[A_\mu]$ are not gauge invariant. Here we should write the color current of quarks j_μ^b

$$j_\mu^b = \bar{\psi}\gamma^\mu T^b \psi. \tag{5.15}$$

Under the following gauge transformation

$$\psi' = (1 - igT^a\chi^a)\psi \tag{5.16}$$

the quark color current j_μ^b changes into

$$j'^b_\mu = \bar{\psi}'\gamma^\mu T^b \psi' = \bar{\psi}(1 + igT^a\chi^a)\gamma^\mu T^b (1 - igT^a\chi^a)\psi \tag{5.17}$$

$$\neq \bar{\psi}\gamma^\mu T^b \psi \tag{5.18}$$

and thus, the quark color current is not invariant under the local gauge transformation. This is quite important since the color charge of quarks should not be physical observables. In fact, the color current of quarks is not conserved.

5.3.2 Quark Confinement

The color charges of quarks should depend on time, and therefore, they are not physical observables. In fact, this is directly connected to the confinement of quarks. This means that quarks are confined kinematically, not dynamically, and therefore, the confinement of quarks must be absolute.

Also, it is not very difficult to prove that the free Lagrangian density of QCD should depend on the gauge. However, it is only recent that this point is realized and confirmed, and this is quite unfortunate indeed.

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