

## 4-2 Hamilton 方程式

Hamilton 方程式の導出  $\iff$  最汎作用の原理

$$S = \int_{t_1}^{t_2} L(q, \dot{q}) dt$$

$$= \int_{t_1}^{t_2} [p\dot{q} - H(p, q)] dt$$

$$\delta S = \int_{t_1}^{t_2} \left[ \delta p \cdot \dot{q} + p \delta \dot{q} - \left( \frac{\partial H}{\partial p} \delta p + \frac{\partial H}{\partial q} \delta q \right) \right] dt$$

$$\text{第2項} = \int_{t_1}^{t_2} p \delta \dot{q} dt = \underbrace{[p \delta q]}_{\substack{0 \\ t_1}}^{t_2} - \int_{t_1}^{t_2} \dot{p} \delta q dt$$

$\delta S = 0$

$$\delta S = \int_{t_1}^{t_2} \left[ \dot{q} \delta p - \dot{p} \delta q - \frac{\partial H}{\partial p} \delta p - \frac{\partial H}{\partial q} \delta q \right] dt = 0$$

(最汎作用の原理)

$\delta p, \delta q$  の係数はゼロ

$$\therefore \boxed{\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}}$$

(Hamilton 方程式)