【レポート問題(5/12)解答】

1. $m\ddot{\mathbf{r}} = \mathbf{F}$ $\quad \begin{cases} m\ddot{x} = F_x \\ m\ddot{y} = F_y \\ m\ddot{z} = F_z \end{cases}$

(a) $\begin{cases} \ddot{x}' = \ddot{x} \\ \ddot{y}' = \ddot{y} \\ \ddot{z}' = \ddot{z} \end{cases}$ よって $\begin{cases} m\ddot{x}' = F_x' \\ m\ddot{y}' = F_y' \\ m\ddot{z}' = F_z' \end{cases}$

(b) $\begin{cases} \ddot{x}' = \ddot{x} \\ \ddot{y}' = \ddot{y} \\ \ddot{z}' = \ddot{z} + 2g \end{cases}$ $\quad \begin{cases} m\ddot{x}' = F_x' \\ m\ddot{y}' = F_y' \\ m(\ddot{z}' - 2g) = F_z' \end{cases}$

これは不変ではない!!

(c) Lorentz 変換

$x' = x, \quad y' = y$

$$z' = \gamma(z + vt)$$
$$t' = \gamma\left(t + \frac{v}{c^2}z\right)$$

$$\dot{x}' = \frac{dx}{dt'} = \frac{1}{\gamma}\frac{1}{\left(1+\frac{v\dot{z}}{c^2}\right)}\frac{dx}{dt} = \frac{\dot{x}}{\gamma\left(1+\frac{v\dot{z}}{c^2}\right)}$$

$$\ddot{x}' = \frac{d\dot{x}'}{dt'} = \frac{1}{\gamma^2\left(1+\frac{v\dot{z}}{c^2}\right)}\frac{d}{dt}\left(\frac{\dot{x}}{1+\frac{v\dot{z}}{c^2}}\right)$$

$$= \frac{1}{\gamma^2\left(1+\frac{v\dot{z}}{c^2}\right)}\left(\frac{\ddot{x}}{1+\frac{v\dot{z}}{c^2}} - \frac{\frac{v}{c^2}\dot{x}\ddot{z}}{\left(1+\frac{v\dot{z}}{c^2}\right)^2}\right)$$

$$\therefore \ddot{x}' = \frac{1}{\gamma^2\left(1+\frac{v\dot{z}}{c^2}\right)^3}\left(\ddot{x} + \frac{v}{c^2}(\ddot{x}\dot{z} - \dot{x}\ddot{z})\right) \neq \ddot{x}$$

$\ddot{y}'$ も同様. 不変ではない.

$$\ddot{z}' = \frac{\ddot{z}}{\gamma^3 \left(1 + \frac{v\dot{z}}{c^2}\right)^3}$$ 不変ではない.

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2. (a) $$\frac{dU(x)}{dx} = -F(x)$$

(b) $$\frac{dU(x)}{dt} = \frac{dU}{dx}\frac{dx}{dt} = -\dot{x}F(x)$$

(c) $$m\ddot{x} = F(x)$$

$$m\dot{x}\ddot{x} = \dot{x}F(x)$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2\right) = -\frac{dU}{dt}$$

$$\therefore \frac{1}{2}m\dot{x}^2 + U = E$$