

[L 1.0-1 問題] 6/30 (略解)

(7)

1. (a) $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

(b) $V = -mgy$

(c) $\begin{cases} x = l \sin \theta \\ y = l \cos \theta \end{cases} \quad \Delta >$

$T = \frac{1}{2} m l^2 \dot{\theta}^2$

(d) $L = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl \cos \theta$

$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} \quad \Delta >$

$m l^2 \ddot{\theta} = -mgl \sin \theta$

$\therefore \ddot{\theta} + \frac{g}{l} \sin \theta = 0$

(e) 小角度近似 $\sin \theta \approx \theta$

$\therefore \ddot{\theta} + \frac{g}{l} \theta = 0, \quad \omega = \sqrt{\frac{g}{l}}$

$\ddot{\theta} + \omega^2 \theta = 0$

$\therefore \theta = A \sin \omega t + B \cos \omega t$

$t=0 \text{ 時 } \theta=0, \dot{\theta} = \omega \theta_0 \quad \Delta >$

$\theta = \theta_0 \sin \omega t$

2. 講義 1-1 参照

3. (a) $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\varphi}^2 + \dot{z}^2)$

(b) $L = T$

(i) r 方向: $m\ddot{r} = mr\dot{\varphi}^2$

$$\therefore m(\ddot{r} - r\dot{\varphi}^2) = 0$$

$$a_r = \ddot{r} - r\dot{\varphi}^2$$

(ii) φ 方向:

$$\frac{d}{dt}(mr^2\dot{\varphi}) = 0$$

$$\therefore 2m\dot{r}\dot{\varphi} + mr^2\ddot{\varphi} = 0$$

$$m(2\dot{r}\dot{\varphi} + r\ddot{\varphi}) = 0$$

$$a_\varphi = 2\dot{r}\dot{\varphi} + r\ddot{\varphi}$$

(iii) z 方向: $a_z = \ddot{z}$

(c) $L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\varphi}^2 + \dot{z}^2) - U(r)$

$$\frac{d}{dt}(m\dot{r}) = -\frac{\partial U}{\partial r} + mr\dot{\varphi}^2$$

$$\frac{d}{dt}(mr^2\dot{\varphi}) = 0, \quad \frac{d}{dt}(m\dot{z}) = 0$$

\therefore $mr^2\dot{\varphi}$, $m\dot{z}$ は $\frac{d}{dt}$ で保存量