[レポート問題] 7/17 (解答)

1. (a) $\dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{\boldsymbol{r}}_1} = \dfrac{\partial L}{\partial \boldsymbol{r}_1}$ より

$$m\ddot{\boldsymbol{r}}_1 = -\frac{\partial U(|\boldsymbol{r}_1-\boldsymbol{r}_2|)}{\partial \boldsymbol{r}_1} - \frac{\partial U(|\boldsymbol{r}_1-\boldsymbol{r}_3|)}{\partial \boldsymbol{r}_1}$$

$$= -\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)}{|\boldsymbol{r}_1-\boldsymbol{r}_2|}\frac{\partial U}{\partial r_{12}} - \frac{(\boldsymbol{r}_1-\boldsymbol{r}_3)}{|\boldsymbol{r}_1-\boldsymbol{r}_3|}\frac{\partial U}{\partial r_{13}}$$

ここで $r_{12}=|\boldsymbol{r}_1-\boldsymbol{r}_2|$, $r_{13}=|\boldsymbol{r}_1-\boldsymbol{r}_3|$, $r_{23}=|\boldsymbol{r}_2-\boldsymbol{r}_3|$ とおいた。

(b) 同様にして

$$m\ddot{\boldsymbol{r}}_2 = -\frac{(\boldsymbol{r}_2-\boldsymbol{r}_1)}{|\boldsymbol{r}_2-\boldsymbol{r}_1|}\frac{\partial U}{\partial r_{12}} - \frac{(\boldsymbol{r}_2-\boldsymbol{r}_3)}{|\boldsymbol{r}_2-\boldsymbol{r}_3|}\frac{\partial U}{\partial r_{23}}$$

$$m\ddot{\boldsymbol{r}}_3 = -\frac{(\boldsymbol{r}_3-\boldsymbol{r}_1)}{|\boldsymbol{r}_3-\boldsymbol{r}_1|}\frac{\partial U}{\partial r_{13}} - \frac{(\boldsymbol{r}_3-\boldsymbol{r}_2)}{|\boldsymbol{r}_3-\boldsymbol{r}_2|}\frac{\partial U}{\partial r_{23}}$$

(b)

$$\frac{d\boldsymbol{P}}{dt} = m(\ddot{\boldsymbol{r}}_1+\ddot{\boldsymbol{r}}_2+\ddot{\boldsymbol{r}}_3)$$

$$= -\frac{1}{r_{12}}\frac{\partial U}{\partial r_{12}}\left[(\boldsymbol{r}_1-\boldsymbol{r}_2)+(\boldsymbol{r}_2-\boldsymbol{r}_1)\right]$$

$$-\frac{1}{r_{23}}\frac{\partial U}{\partial r_{23}}\left[(\boldsymbol{r}_2-\boldsymbol{r}_3)+(\boldsymbol{r}_3-\boldsymbol{r}_2)\right]$$

$$-\frac{1}{r_{13}}\frac{\partial U}{\partial r_{13}}\left[(\boldsymbol{r}_1-\boldsymbol{r}_3)+(\boldsymbol{r}_3-\boldsymbol{r}_1)\right]$$

$$\therefore \quad \frac{d\boldsymbol{P}}{dt} = 0$$

2. (a) $$\frac{dL(q,\dot{q})}{dt} = \frac{\partial L}{\partial q}\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q}$$

Lagrange の式より $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$ より

$$\frac{dL}{dt} = \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right)\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q}$$

$$= \frac{d}{dt}\left[\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q}\right]$$

$$\therefore \quad \frac{d}{dt}\left(L - \frac{\partial L}{\partial \dot{q}}\dot{q}\right) = 0$$

より, $L - \frac{\partial L}{\partial \dot{q}}\dot{q} =$ 定数

→ $T = f(q)\dot{q}^2$ より

$$\frac{\partial T}{\partial \dot{q}} = 2 f(q)\dot{q}$$

より, $\dot{q}\frac{\partial T}{\partial \dot{q}} = 2f(q)\dot{q}^2 = 2T$

これより $L - \frac{\partial L}{\partial \dot{q}}\dot{q} = T - U - 2T = \text{const.}$

$$\therefore \quad \underline{T + U = E}$$

↑ 定数

(b) $$\frac{d}{dt} L(q, \dot{q}, t) = \frac{\partial L}{\partial q} \dot{q} + \frac{\partial L}{\partial \dot{q}} \ddot{q} + \frac{\partial L}{\partial t}$$

(a)と同様にして

$$\frac{d}{dt}\left(L - \frac{\partial L}{\partial \dot{q}} \dot{q}\right) = \frac{\partial L}{\partial t}$$

$L - \frac{\partial L}{\partial \dot{q}} \dot{q} = -E$, $\frac{\partial L}{\partial t} = g(t)$ とおくと

$$\frac{dE}{dt} = g(t)$$

力学的エネルギー E は

時間 t に依存する。