1.

(a) $y = f(x)$ Δx $\Delta y \equiv f(x + \Delta x) - f(x)$

$$f'(x) \equiv \frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(b) _____

$$(x^\alpha)' = \alpha x^{\alpha - 1} \qquad\qquad (\alpha : \qquad\qquad)$$

(c) _____ *** (   ) *** $y = f(x(t))$

$$\frac{df(x(t))}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \equiv \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta x} \frac{\Delta x}{\Delta t} = \frac{dy}{dx} \frac{dx}{dt}$$

$$\Delta y \equiv f(x(t + \Delta t)) - f(x(t)) = f(x + \Delta x) - f(x), \quad \Delta x \equiv x(t + \Delta t) - x(t)$$

(d) _____ $f(x) = (ax + b)^\alpha$

$$f'(x) = a\alpha(ax + b)^{\alpha - 1}$$

(e) Taylor $n! \equiv 1 \times 2 \times \cdots \times n$

$$f(x + h) = f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \ldots + \frac{1}{n!}f^{(n)}(x)h^n + ..$$

$x = 0$

$$f(h) = f(0) + f'(0)h + \frac{1}{2}f''(0)h^2 + \ldots + \frac{1}{n!}f^{(n)}(0)h^n + ..$$

1

2.

(a) $f(x, y)$

$$\frac{\partial f(x, y)}{\partial x} \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} \qquad (y \qquad x \qquad)$$

$$\frac{\partial f(x, y)}{\partial y} \equiv \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y} \qquad (x \qquad y \qquad)$$

(b) $f(x(t), y(t), t)$

$$\Delta x \equiv x(t + \Delta t) - x(t), \quad \Delta y \equiv y(t + \Delta t) - y(t)$$

$$\Delta t \to 0 \qquad \Delta x \to 0, \quad \Delta y \to 0$$

$$\frac{df(x(t), y(t), t)}{dt} \equiv \lim_{\Delta t \to 0} \frac{f(x(t + \Delta t), y(t + \Delta t), t + \Delta t) - f(x(t), y(t), t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left\{ \frac{f(x(t + \Delta t), y(t + \Delta t), t + \Delta t) - f(x(t), y(t + \Delta t), t + \Delta t)}{\Delta t} \right.$$

$$+ \frac{f(x(t), y(t + \Delta t), t + \Delta t) - f(x(t), y(t), t + \Delta t)}{\Delta t}$$

$$\left. + \frac{f(x(t), y(t), t + \Delta t) - f(x(t), y(t), t)}{\Delta t} \right\}$$

$$= \lim_{\Delta t \to 0} \left\{ \frac{f(x(t + \Delta t), y(t + \Delta t), t + \Delta t) - f(x(t), y(t + \Delta t), t + \Delta t)}{\Delta x} \frac{\Delta x}{\Delta t} \right.$$

$$+ \frac{f(x(t), y(t + \Delta t), t + \Delta t) - f(x(t), y(t), t + \Delta t)}{\Delta y} \frac{\Delta y}{\Delta t}$$

$$\left. + \frac{f(x(t), y(t), t + \Delta t) - f(x(t), y(t), t)}{\Delta t} \right\}$$

$$= \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial t}$$

3.

$$\boldsymbol{a} = (a_x, a_y, a_z), \qquad \boldsymbol{b} = (b_x, b_y, b_z)$$

・
$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

・

$$\boldsymbol{a} \cdot \boldsymbol{b} \equiv a_x b_x + a_y b_y + a_z b_z = |\boldsymbol{a}||\boldsymbol{b}| \cos \theta$$

・

$$\boldsymbol{a} \cdot \boldsymbol{b} = 0 \qquad \boldsymbol{a} \qquad \boldsymbol{b}$$

・

$$\boldsymbol{a} \times \boldsymbol{b} \equiv (a_y b_z - a_z b_y, \ a_z b_x - a_x b_z, \ a_x b_y - a_y b_x) = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = -\boldsymbol{b} \times \boldsymbol{a}$$

$$|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \sin \theta, \qquad \boldsymbol{a} \times \boldsymbol{a} = 0$$

・   :

$$\boldsymbol{a} \times \boldsymbol{a} = 0$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \mid \boldsymbol{a} \mid\mid \boldsymbol{b} \mid \cos \theta$$

$$\mid \boldsymbol{a} \times \boldsymbol{b} \mid = \mid \boldsymbol{a} \mid\mid \boldsymbol{b} \mid \sin \theta$$

$$(\boldsymbol{a} \times \boldsymbol{b})^2 = \mid \boldsymbol{a} \mid^2 \mid \boldsymbol{b} \mid^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$$

$$|\boldsymbol{a} + \boldsymbol{b}| = \sqrt{\mid \boldsymbol{a} \mid^2 + \mid \boldsymbol{b} \mid^2 + 2 \mid \boldsymbol{a} \mid\mid \boldsymbol{b} \mid \cos \theta}$$

$$\cdot \qquad : \quad \boldsymbol{n} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

(1) $\quad \boldsymbol{e}_x, \quad \boldsymbol{e}_y, \quad \boldsymbol{e}_z$

$$\boldsymbol{e}_x = (1, 0, 0), \qquad \boldsymbol{e}_y = (0, 1, 0) \qquad \boldsymbol{e}_z = (0, 0, 1)$$

$$\begin{cases} |\boldsymbol{e}_x| = 1, \quad |\boldsymbol{e}_y| = 1, \quad |\boldsymbol{e}_z| = 1 \\ \boldsymbol{e}_x \cdot \boldsymbol{e}_y = 0, \quad \boldsymbol{e}_y \cdot \boldsymbol{e}_z = 0, \quad \boldsymbol{e}_z \cdot \boldsymbol{e}_x = 0 \\ \boldsymbol{e}_x \times \boldsymbol{e}_y = \boldsymbol{e}_z, \quad \boldsymbol{e}_y \times \boldsymbol{e}_z = \boldsymbol{e}_x, \quad \boldsymbol{e}_z \times \boldsymbol{e}_x = \boldsymbol{e}_y \end{cases}$$

(2) $\quad \boldsymbol{e}_r, \quad \boldsymbol{e}_\varphi, \quad \boldsymbol{e}_z$

$$\begin{cases} |\boldsymbol{e}_r| = 1, \quad |\boldsymbol{e}_\varphi| = 1, \quad |\boldsymbol{e}_z| = 1 \\ \boldsymbol{e}_r = \cos\varphi \; \boldsymbol{e}_x + \sin\varphi \; \boldsymbol{e}_y \\ \boldsymbol{e}_\varphi = -\sin\varphi \; \boldsymbol{e}_x + \cos\varphi \; \boldsymbol{e}_y \end{cases}$$

$$\begin{cases} \boldsymbol{e}_r \cdot \boldsymbol{e}_\varphi = 0, \quad \boldsymbol{e}_\varphi \cdot \boldsymbol{e}_z = 0, \quad \boldsymbol{e}_z \cdot \boldsymbol{e}_r = 0 \\ \boldsymbol{e}_r \times \boldsymbol{e}_\varphi = \boldsymbol{e}_z, \quad \boldsymbol{e}_\varphi \times \boldsymbol{e}_z = \boldsymbol{e}_r, \quad \boldsymbol{e}_z \times \boldsymbol{e}_r = \boldsymbol{e}_\varphi \end{cases}$$

(3) $\quad \boldsymbol{e}_r, \quad \boldsymbol{e}_\theta, \quad \boldsymbol{e}_\varphi$

$$\boldsymbol{e}_r = \frac{\boldsymbol{r}}{r}$$

$$\begin{cases} |\boldsymbol{e}_r| = 1, \quad |\boldsymbol{e}_\theta| = 1, \quad |\boldsymbol{e}_\varphi| = 1 \\ \boldsymbol{e}_r = \sin\theta \cos\varphi \; \boldsymbol{e}_x + \sin\theta \sin\varphi \; \boldsymbol{e}_y + \cos\theta \; \boldsymbol{e}_z \\ \boldsymbol{e}_\theta = \cos\theta \cos\varphi \; \boldsymbol{e}_x + \cos\theta \sin\varphi \; \boldsymbol{e}_y - \sin\theta \; \boldsymbol{e}_z \\ \boldsymbol{e}_\varphi = -\sin\varphi \; \boldsymbol{e}_x + \cos\varphi \; \boldsymbol{e}_y \end{cases}$$

$$\begin{cases} \boldsymbol{e}_x = \sin\theta \cos\varphi \; \boldsymbol{e}_r + \cos\theta \cos\varphi \; \boldsymbol{e}_\theta - \sin\varphi \; \boldsymbol{e}_\varphi \\ \boldsymbol{e}_y = \sin\theta \sin\varphi \; \boldsymbol{e}_r + \cos\theta \sin\varphi \; \boldsymbol{e}_\theta + \cos\varphi \; \boldsymbol{e}_\varphi \\ \boldsymbol{e}_z = \cos\theta \; \boldsymbol{e}_r - \sin\theta \; \boldsymbol{e}_\theta \end{cases}$$

$$\begin{cases} \boldsymbol{e}_r \cdot \boldsymbol{e}_\theta = 0, \quad \boldsymbol{e}_\theta \cdot \boldsymbol{e}_\varphi = 0, \quad \boldsymbol{e}_\varphi \cdot \boldsymbol{e}_r = 0 \\ \boldsymbol{e}_r \times \boldsymbol{e}_\theta = \boldsymbol{e}_\varphi, \quad \boldsymbol{e}_\theta \times \boldsymbol{e}_\varphi = \boldsymbol{e}_r, \quad \boldsymbol{e}_\varphi \times \boldsymbol{e}_r = \boldsymbol{e}_\theta \end{cases}$$

· **A** :

$$\begin{cases} \boldsymbol{A} = A_x \ \boldsymbol{e}_x + A_y \ \boldsymbol{e}_y + A_z \ \boldsymbol{e}_z \\ \boldsymbol{A} = A_r \ \boldsymbol{e}_r + A_\theta \ \boldsymbol{e}_\theta + A_\varphi \ \boldsymbol{e}_\varphi \end{cases}$$

$\boldsymbol{e}_x, \ \boldsymbol{e}_y, \ \boldsymbol{e}_z \qquad \boldsymbol{e}_r, \ \boldsymbol{e}_\theta, \ \boldsymbol{e}_\varphi$

$$\begin{cases} A_r = A_x \sin\theta \cos\varphi + A_y \sin\theta \sin\varphi + A_z \cos\theta \\ A_\theta = A_x \cos\theta \cos\varphi + A_y \cos\theta \sin\varphi - A_z \sin\theta \\ A_\varphi = -A_x \sin\varphi + A_y \cos\varphi \end{cases}$$

· :

1.

$$\dot{\boldsymbol{e}}_x = \dot{\boldsymbol{e}}_y = \dot{\boldsymbol{e}}_z = 0$$

2.

$$\begin{cases} \dot{\boldsymbol{e}}_r = \dot{\varphi} \ \boldsymbol{e}_\varphi \\ \dot{\boldsymbol{e}}_\varphi = -\dot{\varphi} \ \boldsymbol{e}_r \end{cases}$$

3.

$$\begin{cases} \dot{\boldsymbol{e}}_r = \dot{\theta} \ \boldsymbol{e}_\theta + \dot{\varphi} \sin\theta \ \boldsymbol{e}_\varphi \\ \dot{\boldsymbol{e}}_\theta = -\dot{\theta} \ \boldsymbol{e}_r + \dot{\varphi} \cos\theta \ \boldsymbol{e}_\varphi \\ \dot{\boldsymbol{e}}_\varphi = -\dot{\varphi} \sin\theta \ \boldsymbol{e}_r - \dot{\varphi} \cos\theta \ \boldsymbol{e}_\theta \end{cases}$$

· $\boldsymbol{r} = r\boldsymbol{e}_r$

$$\dot{\boldsymbol{r}} = \dot{r} \ \boldsymbol{e}_r + r\dot{\theta} \ \boldsymbol{e}_\theta + r\dot{\varphi} \sin\theta \ \boldsymbol{e}_\varphi$$

$$\begin{cases} \ddot{\boldsymbol{r}} = a_r \ \boldsymbol{e}_r + a_\theta \ \boldsymbol{e}_\theta + a_\varphi \ \boldsymbol{e}_\varphi \\ a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2 \sin^2\theta \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2 \sin\theta \cos\theta \\ a_\varphi = r\ddot{\varphi} \sin\theta + 2\dot{r}\dot{\varphi} \sin\theta + 2r\dot{\theta}\dot{\varphi} \cos\theta \\ \qquad = \dfrac{1}{r\sin\theta} \dfrac{d}{dt}(r^2\dot{\varphi}\sin^2\theta) \end{cases}$$

4.

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

Euler

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\begin{cases} \cos\theta = \dfrac{1}{2}(e^{i\theta} + e^{-i\theta}) \\[2mm] \sin\theta = \dfrac{1}{2i}(e^{i\theta} - e^{-i\theta}) \end{cases}$$

5.

$$\begin{cases} \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \\[2mm] \tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y} \end{cases}$$

$$a\sin\theta + b\cos\theta = \sqrt{a^2 + b^2}\sin(\theta + \alpha)$$

$$= \sqrt{a^2 + b^2}(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$\left(\cos\alpha = \frac{a}{\sqrt{a^2 + b^2}}, \quad \sin\alpha = \frac{b}{\sqrt{a^2 + b^2}} \right)$$

$$\begin{cases} \sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2} \\[2mm] \sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2} \\[2mm] \cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2} \\[2mm] \cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2} \end{cases}$$

$$\begin{cases} \sin A \sin B = -\dfrac{1}{2}\left[\cos(A+B) - \cos(A-B)\right] \\[2mm] \sin A \cos B = \dfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right] \\[2mm] \cos A \sin B = \dfrac{1}{2}\left[\sin(A+B) - \sin(A-B)\right] \\[2mm] \cos A \cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \end{cases}$$

$$\begin{cases} \sin 2\theta = 2\sin\theta\cos\theta \\[2mm] \sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta) \\[2mm] \cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta) \end{cases}$$

Taylor

$$\begin{cases} \sin x = x - \dfrac{1}{3!}x^3 + \dfrac{1}{5!}x^5 - \dfrac{1}{7!}x^7 + \ldots \\[2mm] \cos x = 1 - \dfrac{1}{2!}x^2 + \dfrac{1}{4!}x^4 - \dfrac{1}{6!}x^6 + \ldots \\[2mm] \tan x = x + \dfrac{1}{3}x^3 + \dfrac{2}{15}x^5 + \dfrac{17}{315}x^7 + \ldots \end{cases}$$

$$\begin{cases} \dfrac{d\sin x}{dx} = \cos x \\[2mm] \dfrac{d\cos x}{dx} = -\sin x \\[2mm] \dfrac{d\tan x}{dx} = \dfrac{1}{\cos^2 x} \end{cases}$$

6.

$$e^x \cdot e^y = e^{(x+y)}, \qquad (e^x)^y = e^{xy}, \qquad e = 2.7182818$$

$$\log xy = \log x + \log y, \qquad \log x^y = y \log x, \qquad \ln x \equiv \log_e x$$

Taylor

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + ... + \frac{1}{n!}x^n + ...$$

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + ...$$

$$\left(\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 + ... \right)$$

$$\begin{cases} \dfrac{de^x}{dx} = e^x \\ \dfrac{d\ln x}{dx} = \dfrac{1}{x} \end{cases}$$