2-3 Solution of Lippmann-Schwinger eq.

$$\psi(\mathbf{r}) = \varphi(\mathbf{r}) - \int \frac{m}{2\pi r} e^{ikr} e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}')\psi(\mathbf{r}')d^3r'$$

$$\Downarrow$$

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{f(\theta)}{r} e^{ikr}$$

$$\therefore \quad \boxed{f(\mathbf{k},\mathbf{k}') \equiv f(\theta) = -\frac{m}{2\pi}\int e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}')\psi(\mathbf{r}')d^3r'}$$

$f(\mathbf{k},\mathbf{k}')$: scattering amplitude.

$$f(\mathbf{k},\mathbf{k}') = -\frac{m}{2\pi} \langle \mathbf{k}' | V | \psi_{\mathbf{k}} \rangle$$

$$\boxed{f(\mathbf{k},\mathbf{k}') = -\frac{m}{2\pi} \langle \mathbf{k}' | T | \mathbf{k} \rangle}$$

T-matrix

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\mathbf{k},\mathbf{k}')|^2}$$

as defined.

2-4 T-matrix from Lippman-Schwinger eq.

Lippman-Schwinger eq.

$$|\psi\rangle = |\varphi\rangle + \frac{1}{E-H_0+i\epsilon} V|\psi\rangle$$

Multiplying V from the left-hand side, we obtain

$$\langle\varphi|V|\psi\rangle = \langle\varphi|V|\varphi\rangle + \langle\varphi|\frac{1}{E-H_0+i\epsilon} V|\psi\rangle$$

◎ Definition of T-matrix

$$T = \langle\varphi|V|\psi\rangle \equiv \langle\varphi|T|\varphi\rangle$$

Thus, we find

$$\langle\varphi|T|\varphi\rangle = \langle\varphi|V|\varphi\rangle$$

$$+ \langle\varphi|V|\varphi\rangle\langle\varphi|\frac{1}{E-H_0+i\epsilon}|\varphi\rangle$$

$$\times \langle\varphi|V|\psi\rangle$$

$$\therefore \boxed{T = V + VGT} \qquad G = \langle\varphi|\frac{1}{E-H_0+i\epsilon}|\varphi\rangle$$