3-2 Interaction Picture

eg.

We consider the same system

$$\underline{H = H_0 + H_I}$$

Let's introduce Ψ_I by

$$\boxed{\Psi_I = e^{iH_0 t}\,\Psi_S}$$

(Schrodinger picture)

$$\therefore \quad i\frac{\partial \Psi_S}{\partial t} = (H_0 + H_I)\,\Psi_S$$

Thus, we find

$$
\begin{aligned}
i\frac{\partial \Psi_S}{\partial t} &= i\frac{\partial}{\partial t}\left(e^{-iH_0 t}\,\Psi_I\right) \\
&= H_0 e^{-iH_0 t}\,\Psi_I + i\,e^{-iH_0 t}\frac{\partial \Psi_I}{\partial t} \\
&= (H_0 + H_I)\,e^{-iH_0 t}\,\Psi_I
\end{aligned}
$$

$$\therefore \quad \boxed{i\frac{\partial \Psi_I}{\partial t} = e^{iH_0 t}\,H_I\,e^{-iH_0 t}\,\Psi_I}$$

- Now, we expand Ψ_I in terms of u_n

$$\left\{ \begin{array}{l} \underline{H_0 u_n = E u_n} \\ \underline{\langle u_n | u_{n'} \rangle = \delta_{nn'}} \end{array} \right.$$

$$i \frac{\partial \Psi_I}{\partial t} = e^{iH_0 t} H_I e^{-iH_0 t} \Psi_I$$

By operating $\langle u_k |$ from the left-hand side, we obtain

$$i \frac{\partial}{\partial t} \langle u_k | \Psi_I \rangle = \sum_n \langle u_k | e^{iH_0 t} | u_k \rangle \times \langle u_k | H_I u_n \rangle$$

$$\times \langle u_n | e^{-iH_0 t} | u_n \rangle \langle u_n | \Psi_I \rangle$$

We define $C_k(t)$ by

$$\boxed{C_k(t) \equiv \langle u_k | \Psi_I \rangle}$$

Thus, we find

$$i \frac{\partial}{\partial t} C_k(t) = \sum_n e^{i(E_k - E_n)t} \langle u_k | H_I | u_n \rangle \times C_n(t)$$

$$\therefore \boxed{i \frac{\partial C_k(t)}{\partial t} = \sum_n \langle u_k | H_I | u_n \rangle e^{i\omega_{kn} t} C_n(t)}$$

with $(\omega_{kn} \equiv E_k - E_n)$.

This is just the same as the non-static perturbation theory.