# 5. Cross Section



## 5-1 Differential cross section

$$\boxed{\sigma_{fi} \equiv \frac{\text{Scattering rate}}{\text{incident flux}}}$$

- Scattering rate :

$$\left\{ \begin{aligned} d\Gamma_{fi} &= \frac{1}{T} |S_{fi}|^2 \, dN_f \\ S_{fi} &= -i\, \delta(E_i - E_f)\, T_{fi} \end{aligned} \right.$$

- incident flux : $\boxed{\frac{p_i}{m}}$ $\quad p_i$ : incident momentum

- Differential cross section : Elastic scattering

$$d\sigma_{fi} = \frac{m}{p_i}\, d\Gamma_{fi}$$

$$= \frac{m}{p_i} \cdot \frac{2\pi}{T} \delta(0)\, \delta(E_f - E_i)\, |T_{fi}|^2 \, dN_f$$

$$= \frac{m}{p_i} \cdot 2\pi \int \delta(E_f - E_i)\, |T_{fi}|^2 \underbrace{\frac{d^3 p_f}{(2\pi)^3}}_{}$$

(one-body final state)

$$\therefore \quad d\sigma_{fi} = \frac{m}{p_i} \frac{1}{(2\pi)^2} \int \delta(E_f - E_i)\; p_f^2 dp_f \, d\Omega \; |T_{fi}|^2$$

$$dE_f = \frac{p_f}{m} dp_f$$

Thus, we find (with $p_i = p_f$ )

$$d\sigma_{fi} = \frac{m}{p_i} \cdot \frac{1}{(2\pi)^2} \cdot \frac{m}{p_f} \cdot p_f^2 \, |T_{fi}|^2 \, d\Omega$$

$$\therefore \; d\sigma_{fi} = \frac{m^2}{(2\pi)^2} \, |T_{fi}|^2 \, d\Omega$$

- On the other hand, we have the relation between scattering amplitude $f$ and T-matrix $T_{fi}$

$$\boxed{f = -\frac{m}{2\pi} T_{fi}}$$

Therefore,

$$\boxed{d\sigma = |f|^2 \, d\Omega}$$